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GNSS Under Attack

GNSS in Aviation & Sensor Integration

Day 1 Lecture – 15:30-16:15

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Pseudorange and Carrier-Phase Error Equations

Pseudorange (PR):

User clock error

Error due to delay in Troposphere

Error due to Multipath (Reflections)

Satellite clock and orbit error

$$PR_j = R_j + c\delta t_u + \delta R_{iono} + \delta R_{tropo} + \delta R_{PR,noise} + \delta R_{PR,mp} + \delta R_{PR,hw} - c\delta t_{SV,j}$$

True geometric range

Error due to delay in Ionosphere

Error due to Thermal Noise

Error due to Hardware Delays in Receiver

Carrier-Phase (CP):

$$\phi_j = R_j + N_j\lambda + c\delta t_u - \delta R_{iono} + \delta R_{tropo} + \delta R_{ID,noise} + \delta R_{ID,mp} + \delta R_{ID,hw} - c\delta t_{SV,j}$$

Unknown offset

Opposite sign from PR

Much smaller noise (mm-level)

Smaller multipath (cm-level)

Correlation Basics

- **Spatial correlation**:
  - How much does a specific error change as a function of spatial separation;
- **Temporal correlation**:
  - How much does a specific error change as a function of time.

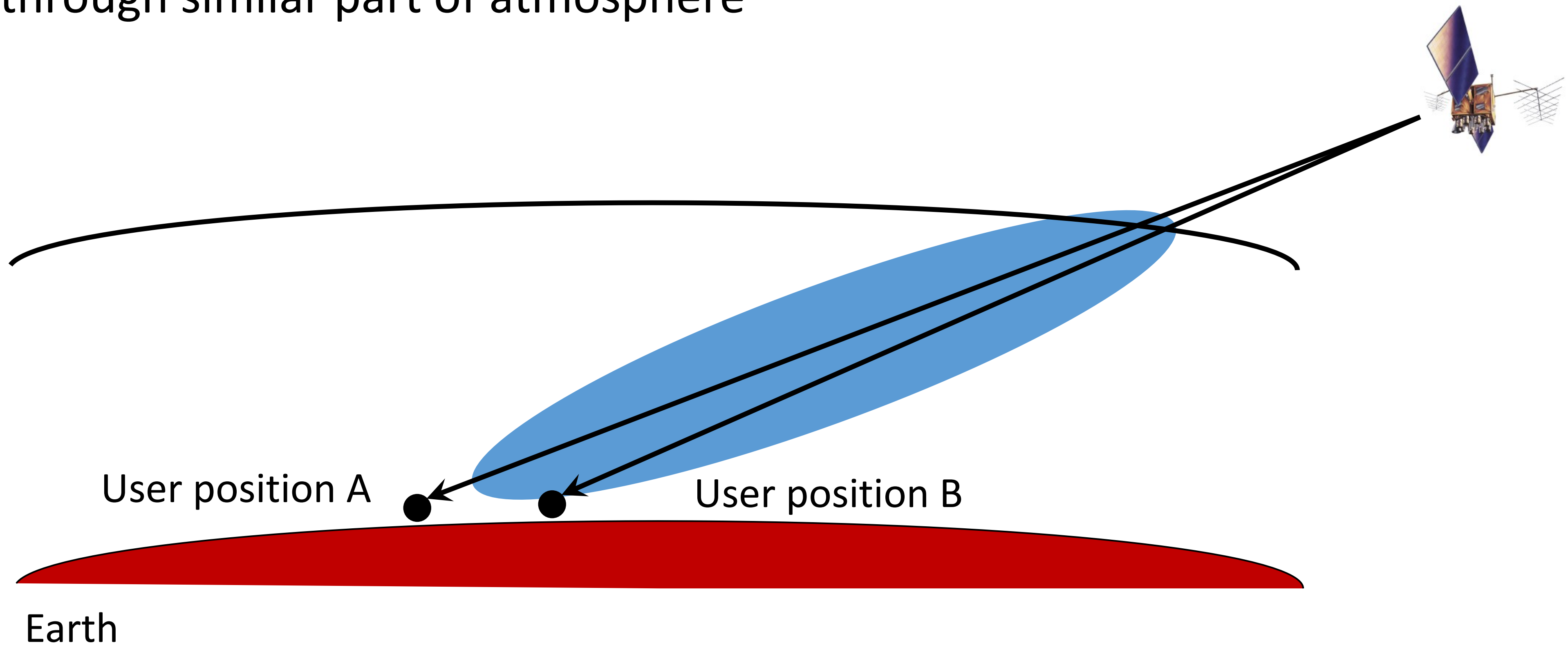
  - For example:

$$\delta PR = \delta R_{iono} + \delta R_{tropo} + \delta R_{noise} + \delta R_{mp} + \delta R_{hw} + \delta R_{SV}$$

Spatially and temporally* uncorrelated

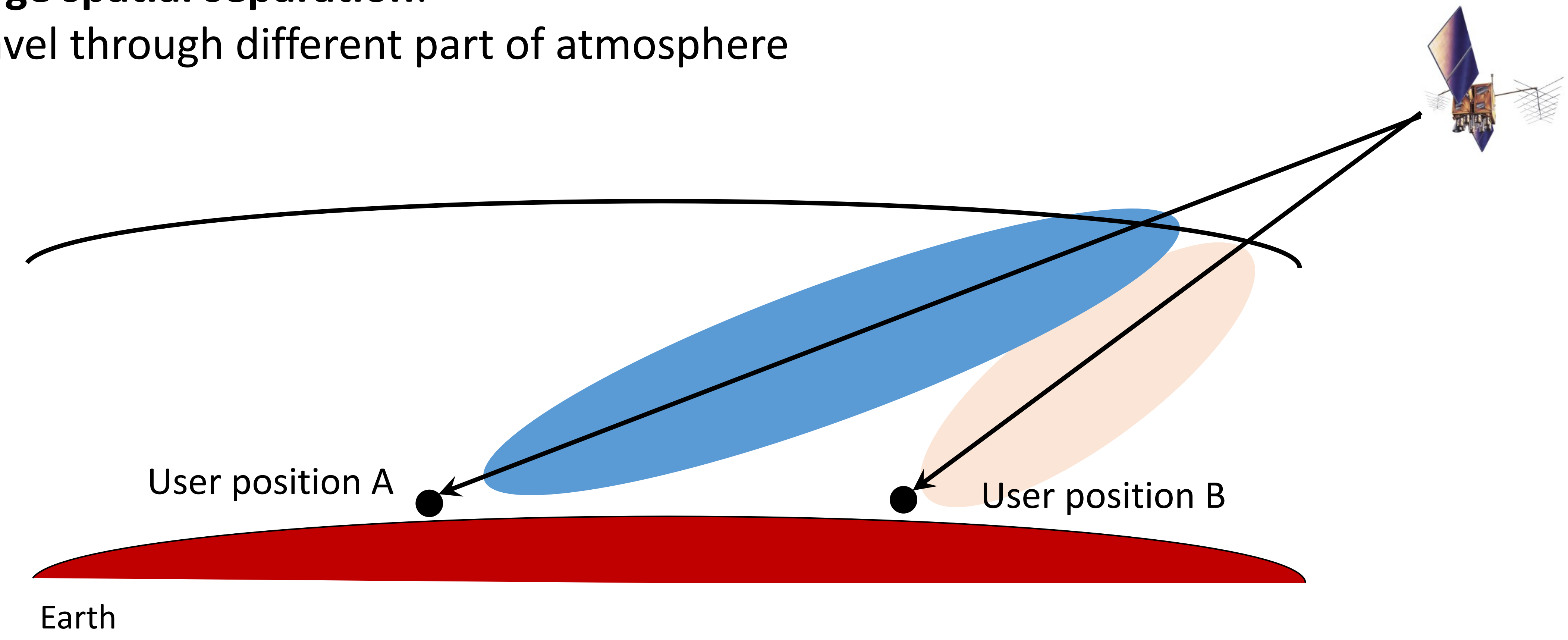
Atmospheric Errors

Small spatial separation:
Travel through similar part of atmosphere



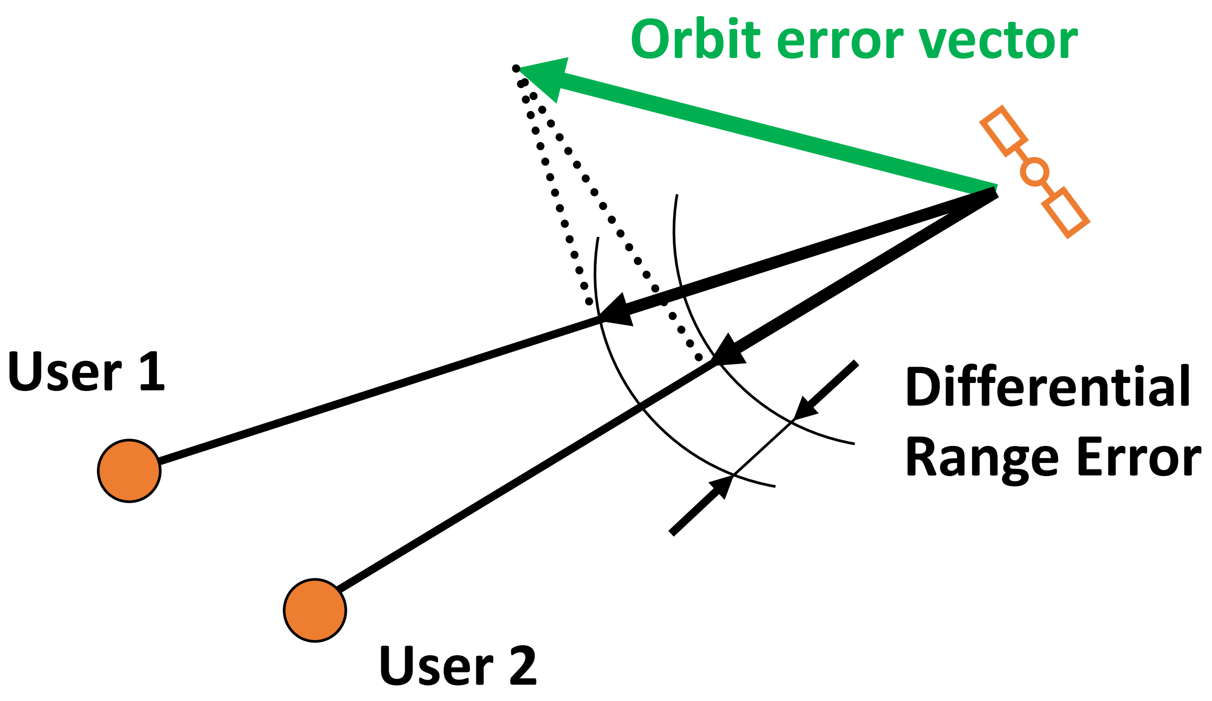
Atmospheric Errors

Large spatial separation:
Travel through different part of atmosphere



Satellite Errors

- **Clock Error**: Is the same in all directions and is therefore common between two receivers.
- **Orbit Error**: Separated users observe different orbit errors.



Various Numbers – Rules-of-Thumb

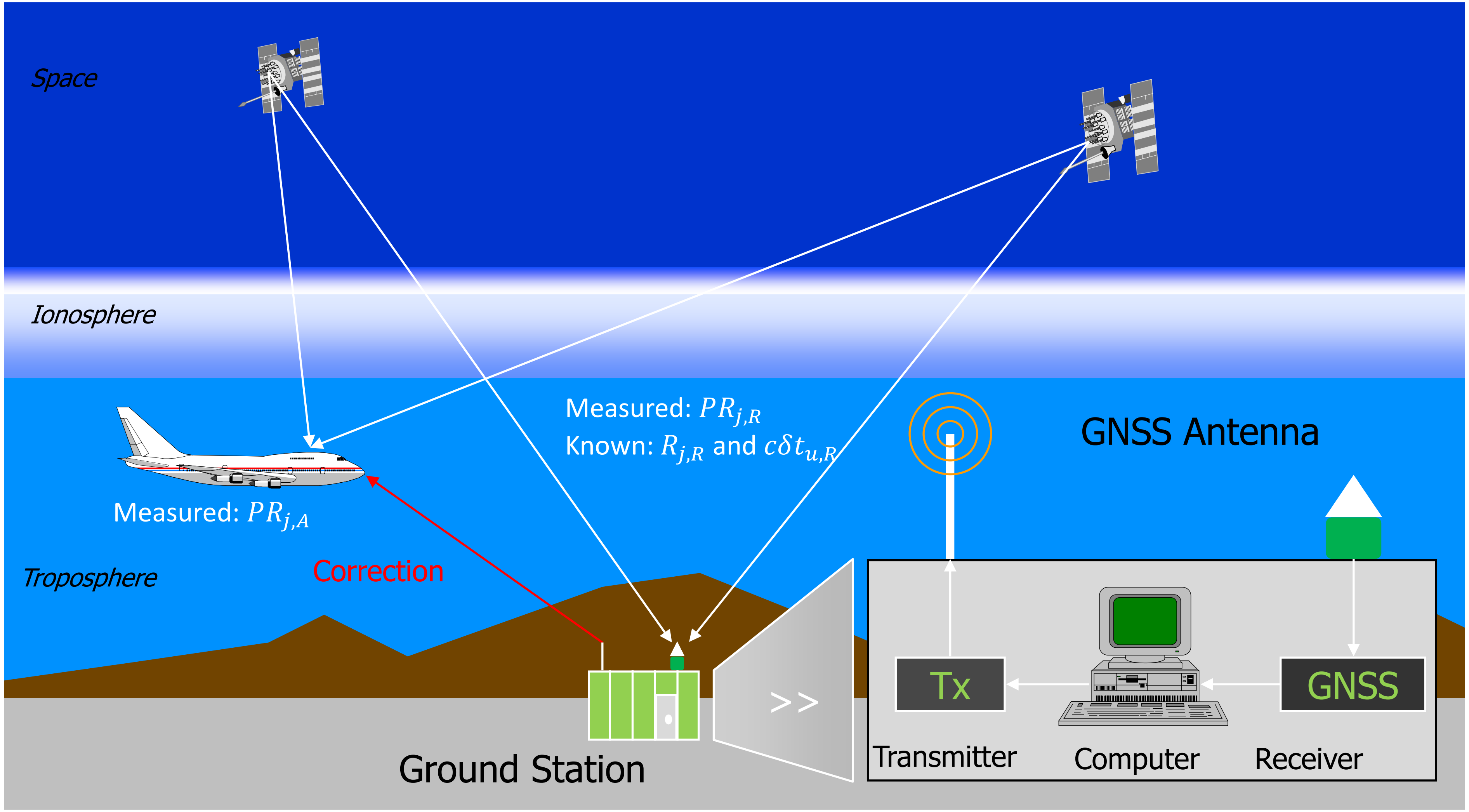
Spatial:	1) Tropospheric Delays	< 0.05 m per 100 km
		< 1 m vertical
	2) Ionospheric Delays	< 0.2 m per 100 km
	3) Satellite Orbit Errors	< 0.5 m per 100 km
Temporal:	4) Changes in 1), 2), 3)	< 0.1 m after 10 s

Note: these are typical numbers, not guaranteed limits.

$$SD_{j,A,B} = PR_{j,A} - PR_{j,B} \approx R_{j,A} - R_{j,B} + c(\delta t_{u,A} - \delta t_{u,B}) + (\delta R_{iono,A} - \delta R_{iono,B}) + (\delta R_{tropo,A} - \delta R_{tropo,B}) + (\delta R_{PR,noise,A} - \delta R_{PR,noise,B}) + (\delta R_{PR,mp,A} - \delta R_{PR,mp,B}) - c(\delta t_{SV,j,A} - \delta t_{SV,j,B})$$

Green: spatially correlated; Red: spatially uncorrelated

Differential GNSS



Potential Solutions to Achieve Performance

- **(European) Technical Standard Orders (TSO)** – GNSS related
  - Define the minimum performance standards for aircraft GNSS equipment including receivers and antennas
- **TSO-C129a/b/c**
  - Stand-alone Airborne Navigation Equipment using the Global Positioning System (GPS) – **May or may not have Fault Detection and Exclusion (FDE)**
- **TSO-C196b**
  - Airborne Supplemental Navigation Sensors for Global Positioning System Equipment using Aircraft-based Augmentation – **Requires Fault Detection and Exclusion (FDE)**
- **TSO-C146e**
  - Stand-alone Airborne Navigation Equipment using the Global Positioning System (GPS) Augmented by the **Satellite Based Augmentation System (SBAS)**
  - See RTCA/DO-229C "Minimum Operational Performance Standards for Global Positioning System/Wide Area Augmentation System Airborne Equipment," Class Gamma or Class Delta-4
- **TSO-C161b**
  - **Ground Based Augmentation System** Positioning and Navigation Equipment
  - See RTCA/DO-253C, Minimum Operational Performance Standards for GPS Local Area Augmentation System Airborne Equipment

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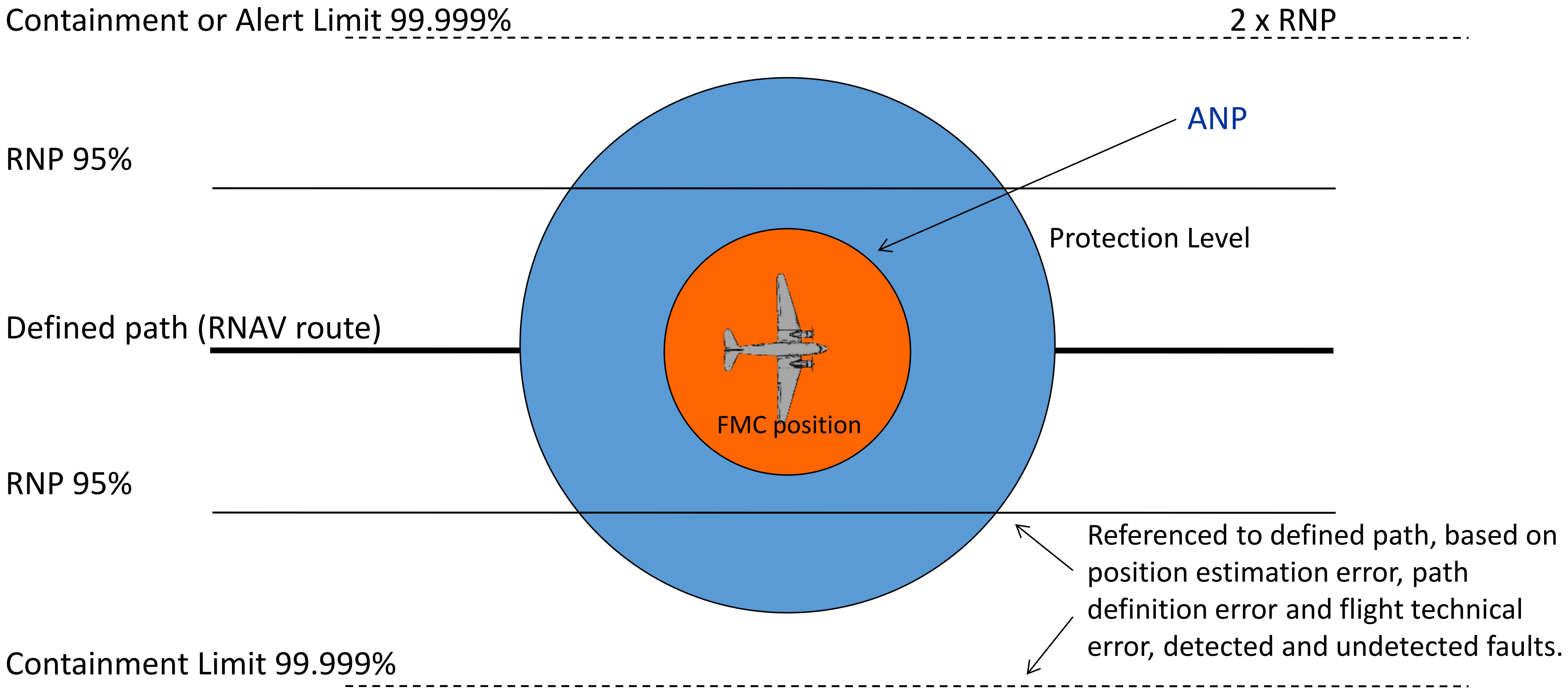
GNSS RNP Procedures



Actual Navigation Performance (ANP):

A measure of the current estimated navigational performance (95% performance). Also referred to as Estimated Position Error (EPE).

GNSS RNP Procedures (En-route and Approach)

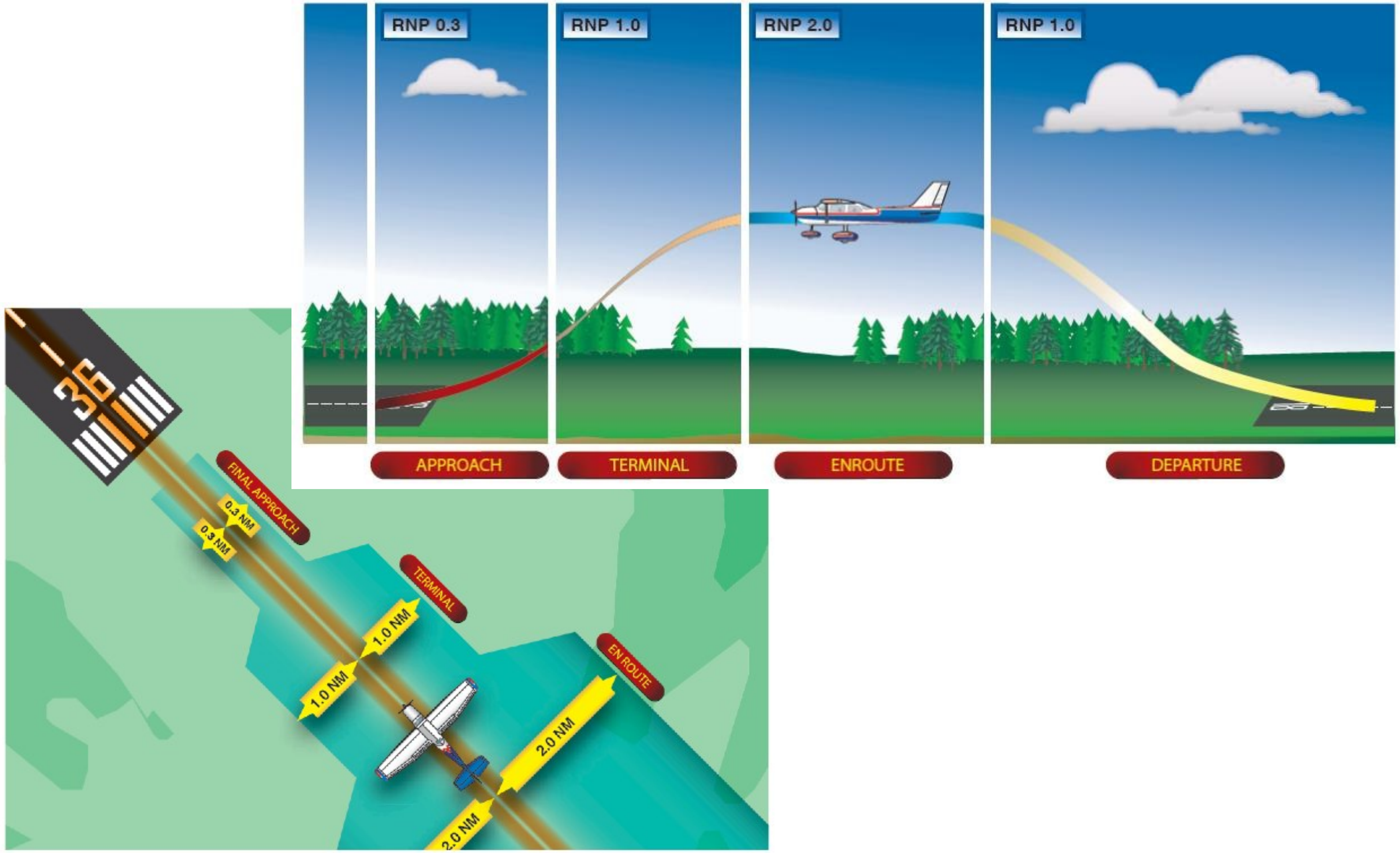
- RNP X (e.g. RNP 1, RNP 0.3)
- Accuracy:
  - The maximum TSE with 95% probability <= 1xRNP
- Integrity:
  - Probability to transgress the containment limit set at 2xRNP without alert must be XXX/flight hour
- Continuity:
  - Probability of RNP capability loss with alert must be <XXX/flight hour
- Availability:
  - Probability that the function is available XXX% of the time

Example: RNP 0.3 Approach Procedure

- What does RNP 0.3 mean?
- Accuracy: 307m 95%
  - Note: RNP specifies Total System Error, which includes the Navigation System Error (NSE) and the Flight Technical Error (FTE). An FTE of 0.25 nmi is assumed for RNP 0.3

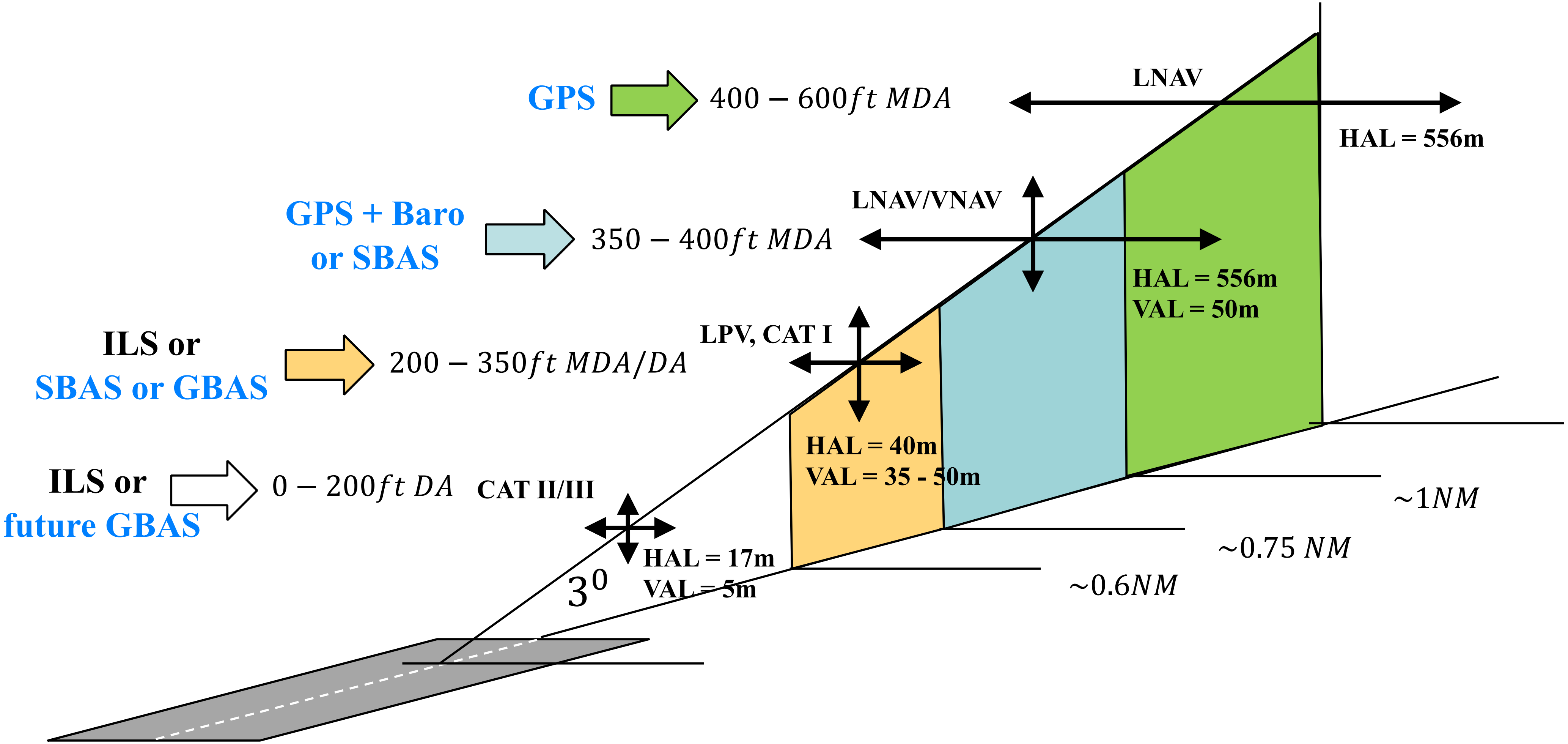
    → $\sqrt{0.3^2 - 0.25^2} = 0.1658\ nmi = 307.1m$
- Integrity: 10^{-5}/hour
  - Probability of hazardous misleading information is 10^{-5}/hour
- Time-To-Alert: 10 sec
- Availability: 99.99%
- Continuity: $1 - 10^{-4}$/hr

RNP during Phases of Flight



From: FAA-H-8083-15B

GPS Approaches

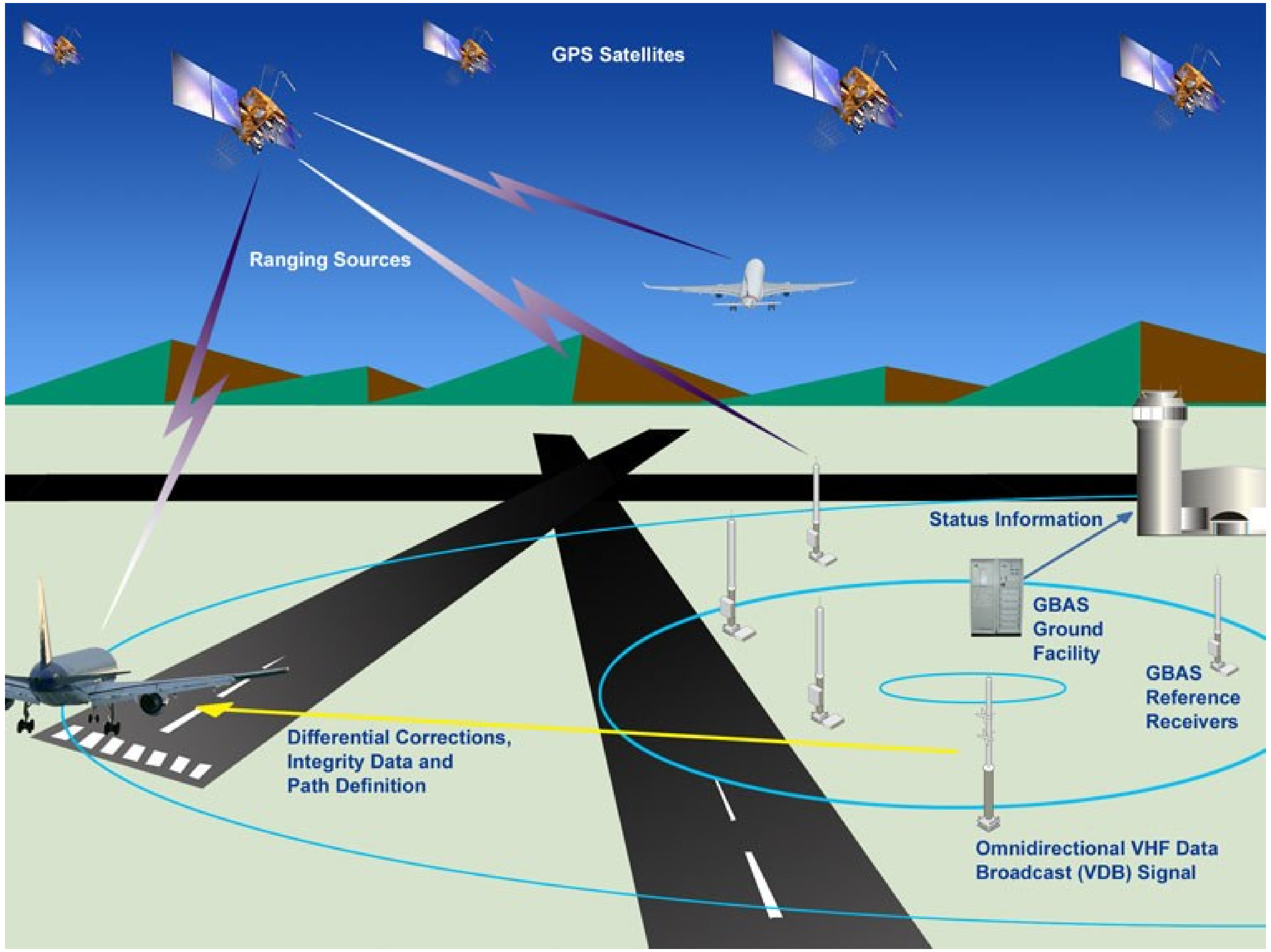


Based on: T. Walter, et al., "Worldwide Vertical Guidance of Aircraft Based on Modernized GPS and New Integrity Augmentations," Proceedings of the IEEE, December 2008.

GNSS Enables Navigation

	Procedure	Hor Acc (95%)	Vert Acc (95%)	HAL	VAL	TTA	Integrity (P_{HMI})	Continuity
Oceanic								
	RNP 4	4 nm	-[1]	8 nm	-[1]	-	10^{-5}/hr	1 - 10^{-4}/hr
En-route								
	RNP 2	2 nm	-[1]	4 nm	-[1]	-	10^{-5}/hr	1 - 10^{-4}/hr
Terminal								
	RNP 1	1 nm	-[1]	2 nm	-[1]	-	10^{-5}/hr	1 - 10^{-4}/hr
Approach								
	RNP 0.3	0.3 nm/556 m	-[1]	0.6 nm	-[1]	-	10^{-5}/hr	1 - 10^{-4}/hr
RNP-APCH								
Non-precision approach (NPA) – See approach plates for minima								
	LNAV[2]	220 m	N/A	556 m	N/A	10 s	10^{-7}/hr	1 - 10^{-5}/hr
*	LP	16 m	N/A	40 m	N/A	6.2 s	$2{\cdot}10^{-7}$/app[3]	
Precision approach (PA) – See approach plates for minima								
*	LNAV/VNAV[2]	220 m	20 m	556 m	50 m	10 s	$2{\cdot}10^{-7}$/app[3]	1 – $5.5{\cdot}10^{-5}$/15s
*	LPV	16 m	20 m	40 m	50 m	6.2 s	$2{\cdot}10^{-7}$/app[3]	1 – $8{\cdot}10^{-5}$/15s
*	LPV200	16 m	4 m	40 m	35 m	6.2 s	$2{\cdot}10^{-7}$/app[3]	1 – $8{\cdot}10^{-5}$/15s
*	GBAS (GAST-C[4])	16 m	4 m	40 m	10 m	6 s	$2{\cdot}10^{-7}$/app[3]	1 – $8{\cdot}10^{-6}$/15s
*	GBAS (GAST-C[5])	15 m	2.9 m	17 m	10 m	2 s	$1{\cdot}10^{-9}$/15 s[6] or 30s[7]	1 – $8{\cdot}10^{-6}$/15s

GBAS – for Final Approach Guidance



From: http://www.faa.gov/about/office_org/headquarters_offices/ato/service_units/techops/navservices/gnss/laas/

Differential GPS architecture

Exploits
(i) the fact that many errors in GPS are **spatially** correlated, like ionospheric and tropospheric delays
(ii) fact that carrier-phase measurement are less noisy that pseudoranges

Much focus on integrity, availability and continuity in addition to accuracy

*a.k.a. LAAS in U.S.A.

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Ground Based Augmentation System (GBAS)

- **Reference Receiver** at a known location
  - Calculates the error

$$PR_{j,R} = R_{j,R} + c\delta t_{u,R} + \delta R_{iono,R} + \delta R_{tropo,R} + \delta R_{noise,R} + \delta R_{mp,R} - c\delta t_{SV,j,R}$$

Known since position of reference station and satellite are known

Calculated

PseudoRange Correction (PRC), PseudoRange Rate (RRC), Tropospheric correction (TC), Satellite Clock Correction (SVC)

- **Datalink**
  - Broadcast the corrections to all aircraft in the coverage area
- **Aircraft**
  - Apply the corrections and most of the correlated errors will be removed

$$PR_{j,A} + PRC_{j,R} + RRC_{j,R}\Delta t + TC + cSVC \approx R_{j,R} + c\delta t_{u,R} + \delta R_{noise,A+R} + \delta R_{mp,A+R}$$

Make sure to keep the multipath from the ground small

Note: slightly simplified

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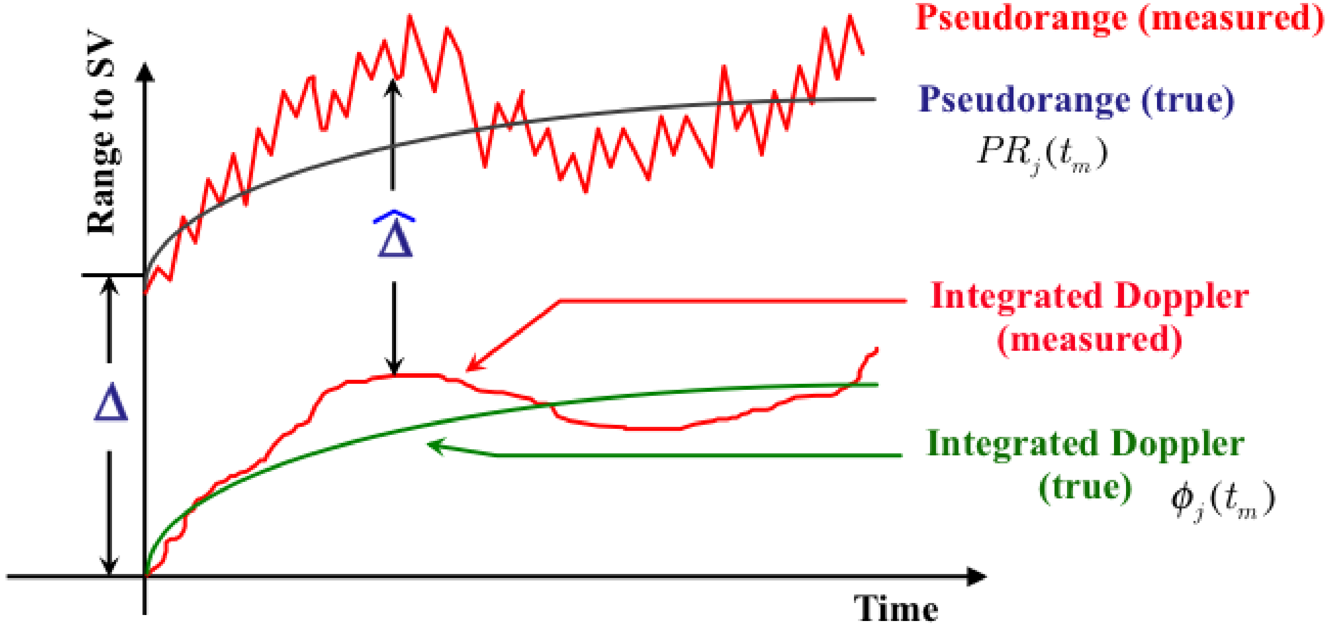
GBAS – Key Design Elements

- **Reduce noise** through carrier-smoothing of pseudoranges
- **Reduce ground multipath** error by using special multipath limiting antennas
- Use **multiple reference antennas** to
  - Perform fault detection
  - Improve accuracy through averaging

- **Integrity Processing**
  - Code Carrier Divergence Monitor (CCDM), Differential Correction Magnitude Check (DCMC), Bias Approach Monitor (BAM), Reference Receiver Fault Monitor (RRFM), Geometry Screening, DSIGMA Monitor

GBAS – Carrier-Smoothed Code

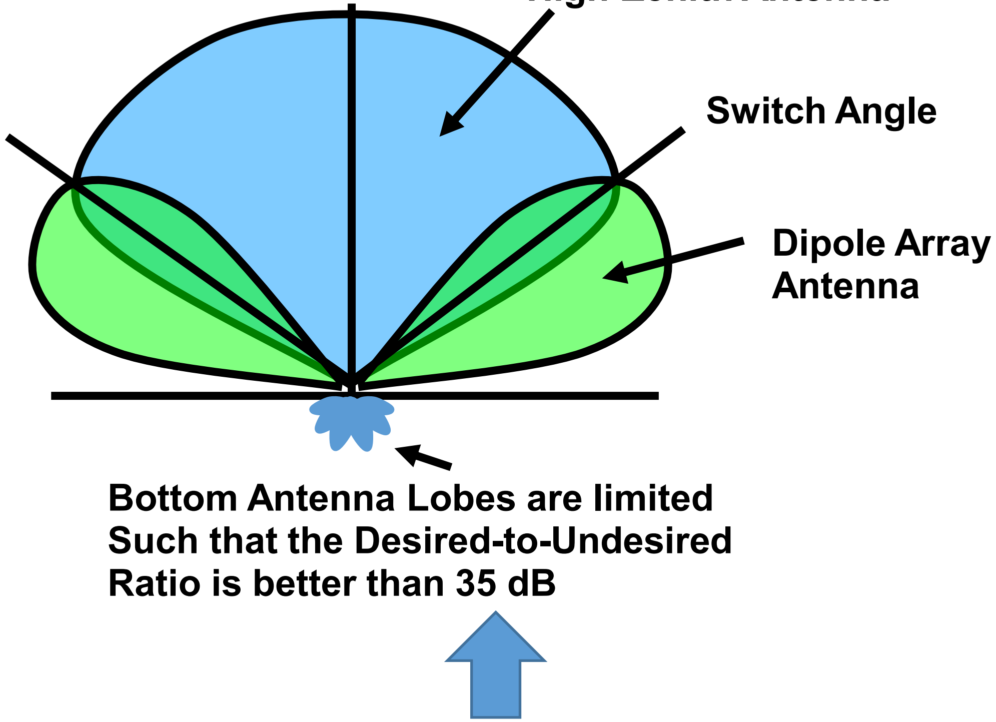
$$PR_{j,smooth}(t_k) = \frac{N-1}{N}\left[PR_{j,smooth}(t_{k-1}) + \phi_j(t_k) - \phi_j(t_{k-1})\right] + \frac{1}{N}PR_j(t_k)$$
$$= \frac{N-1}{N}\left[PR_{j,smooth}(t_{k-1}) + \Delta\phi_j\right] + \frac{1}{N}PR_j(t_k)$$



- $PR_{j,smooth}$ **is the smoothed pseudorange**
- $PR_{j,meas}$ **is the measured pseudorange**
- $\Delta\phi_j$ **is the change in the accumulated carrier phase**
- **N is the total number of measurements**
  - **For a 1-s update rate, N = 100 for a smoothing time of 100s**

$$\Rightarrow \quad \sigma_{steady-state} = \frac{\sigma_{meas}}{\sqrt{2N}}$$

GBAS – Multipath Reduction



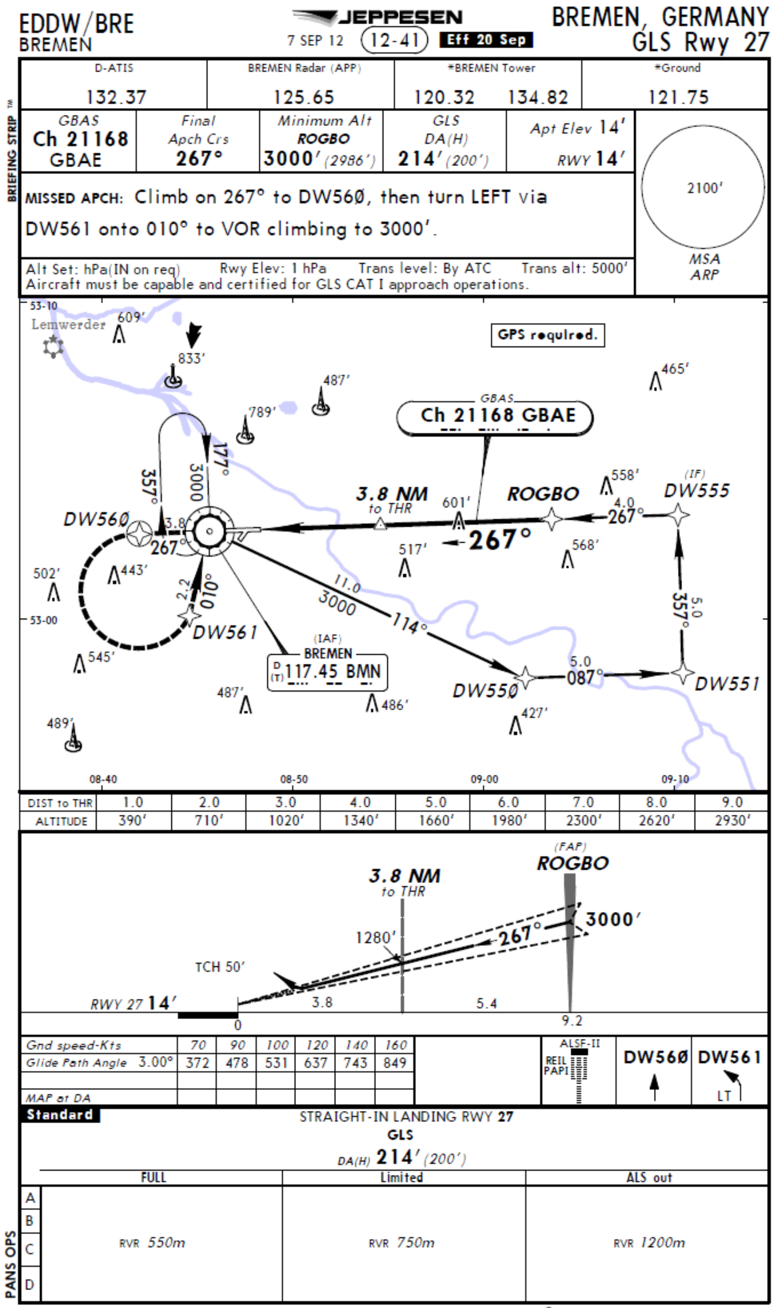
In that way the pseudorange multipath error is limited to about 0.1 m

GNSS Augmentations

GPS Landing System (GLS)

Bremen:

First Certified CAT-I GBAS

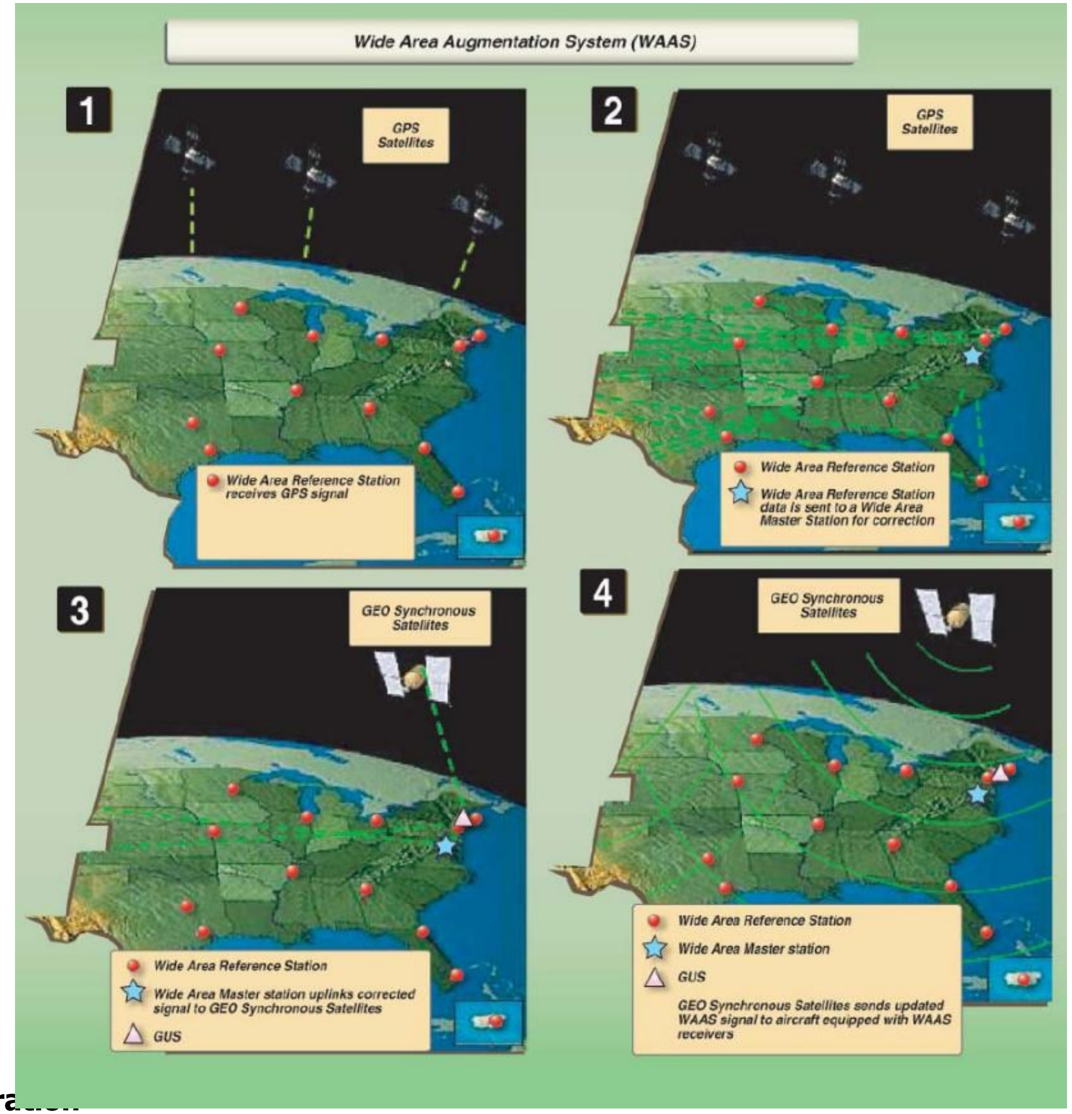


GBAS

THALES

Honeywell SmartPath Precision Approach System

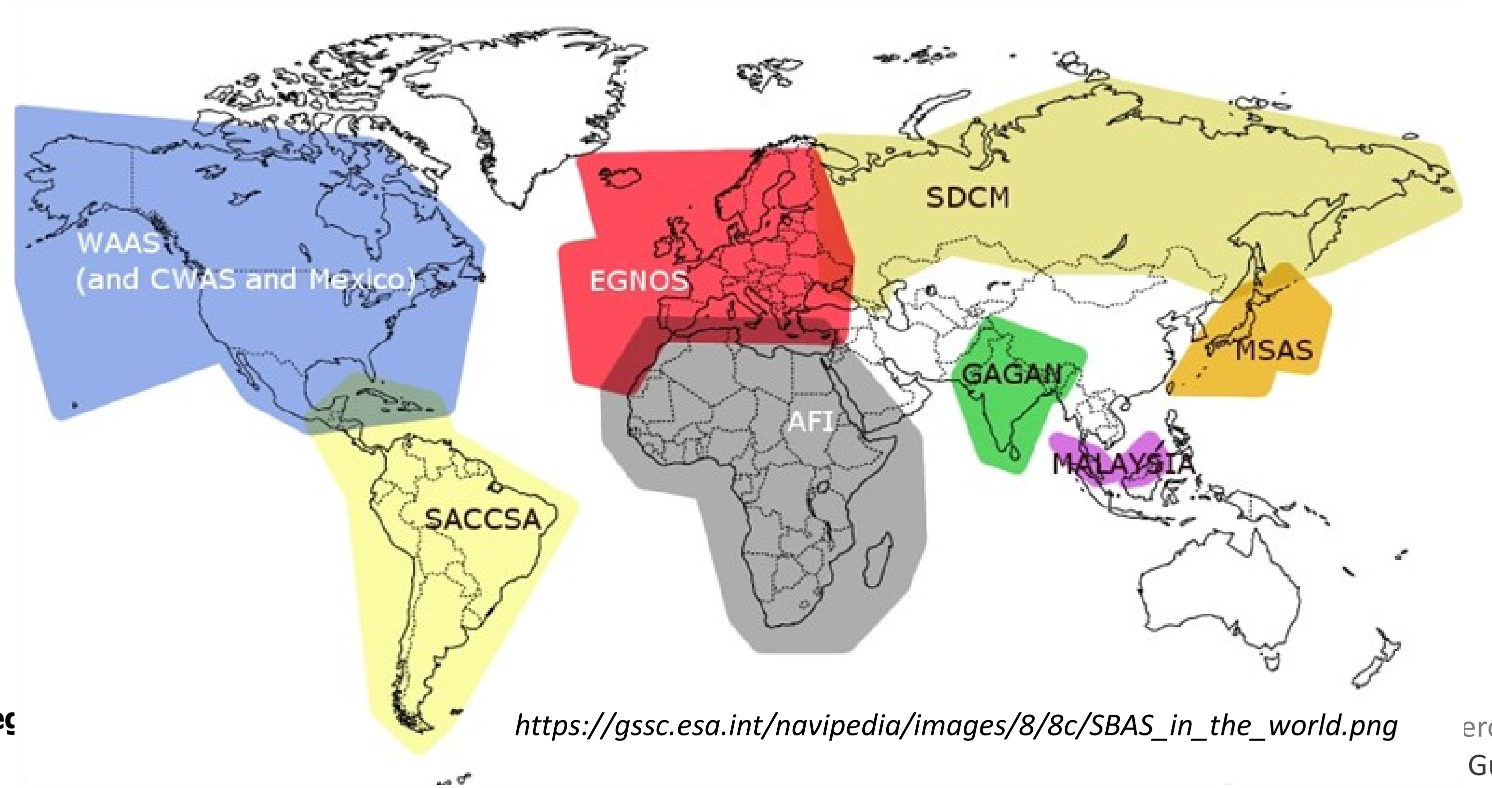
Space Based Augmentation System (SBAS)



- SBAS Reference Stations
- SBAS Master Stations
- GEO Communications Systems
- Transmit a GNSS-like signal (now 200 navigation bits per second instead of 50 bps)

Space Based Augmentation System (SBAS)

- Wide Area Augmentation System (WAAS)
- European Geostationary Navigation Overlay Service (EGNOS)
- Multi-functional Satellite Augmentation System (MSAS)
- GPS Aided Geo Augmented Navigation or GPS and Geo Augmented Navigation system (GAGAN)



https://gssc.esa.int/navipedia/images/8/8c/SBAS_in_the_world.png

SBAS in Europe: EGNOS

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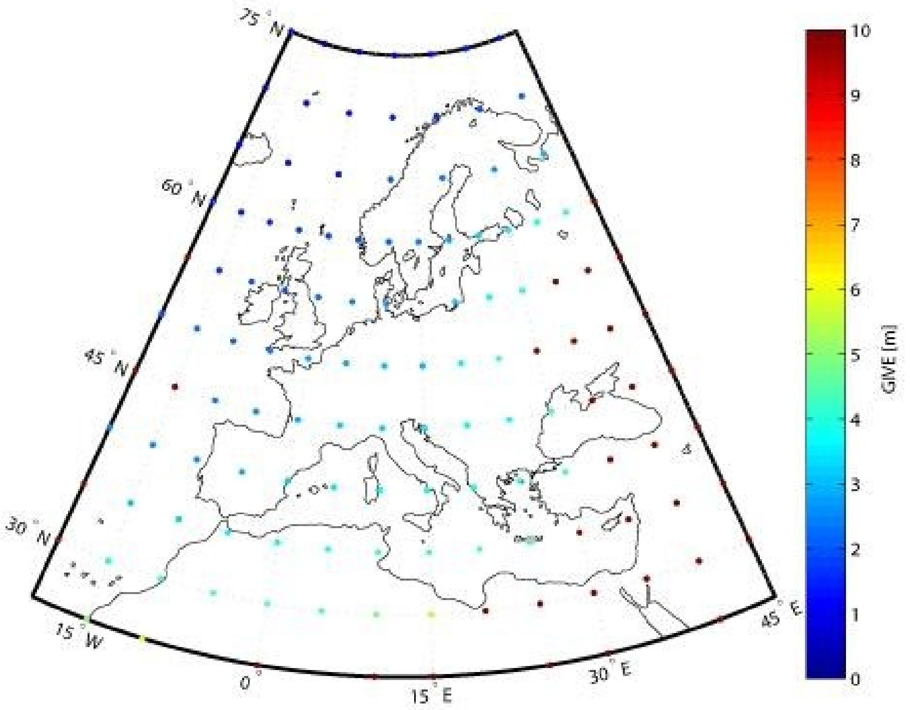
http://egnos-user-support.essp-sas.eu/egnos_ops/sites/all/images/SYSMAPESSP_light.jpg

- 40 Ranging and Integrity Monitoring Stations (RIMS)
- Geostationary Satellites

SBAS Correction

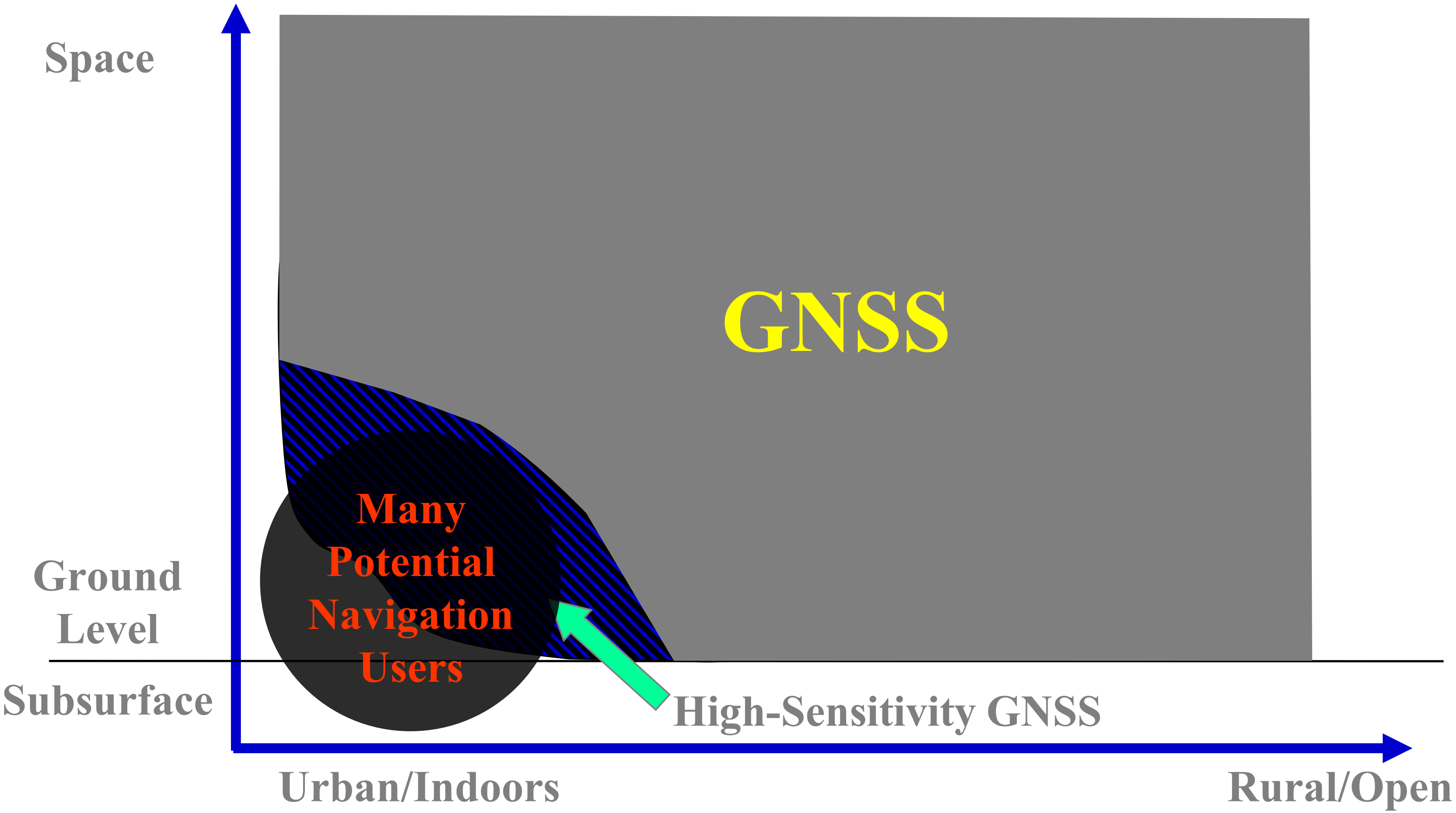
- Downlink provides corrections to ranging signal and satellite position
- There are fast corrections, long term corrections and ionosphere corrections
  - **Fast Corrections** (FCs) for each GPS satellite clock's rapid, short-term errors; calculated and broadcast every 6 seconds for each satellite,
  - **Long Term Corrections** (LTCs) for each GPS satellite clock's slow drift errors and slow ephemeris errors; calculated every 256 seconds and broadcast at least every 120 seconds
  - **Ionosphere Correction** via grid interpolation



Role of GNSS

- What about GNSS? Hasn't GNSS already solved all the problems?
  - GNSS has demonstrated the incredible value of reliable, precision navigation
  - GNSS is the correct solution for many applications
  - But GNSS is not a complete solution:
    - Underground / cave navigation / under trees
    - Navigation in buildings / heavy urban environments
    - Navigation in presence of GNSS jamming (AJ can mitigate but not remove the jamming problem)
    - Surveillance, tracking, targeting, collision avoidance.

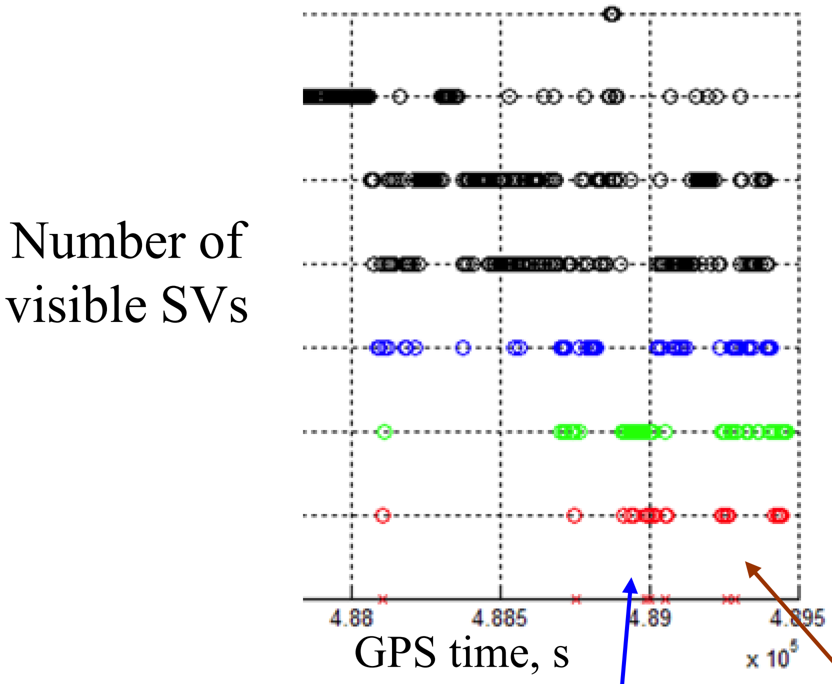
The Navigation Gap



Courtesy of Dr. John Raquet, AFIT and Dr. Mikel M. Miller, IS4S

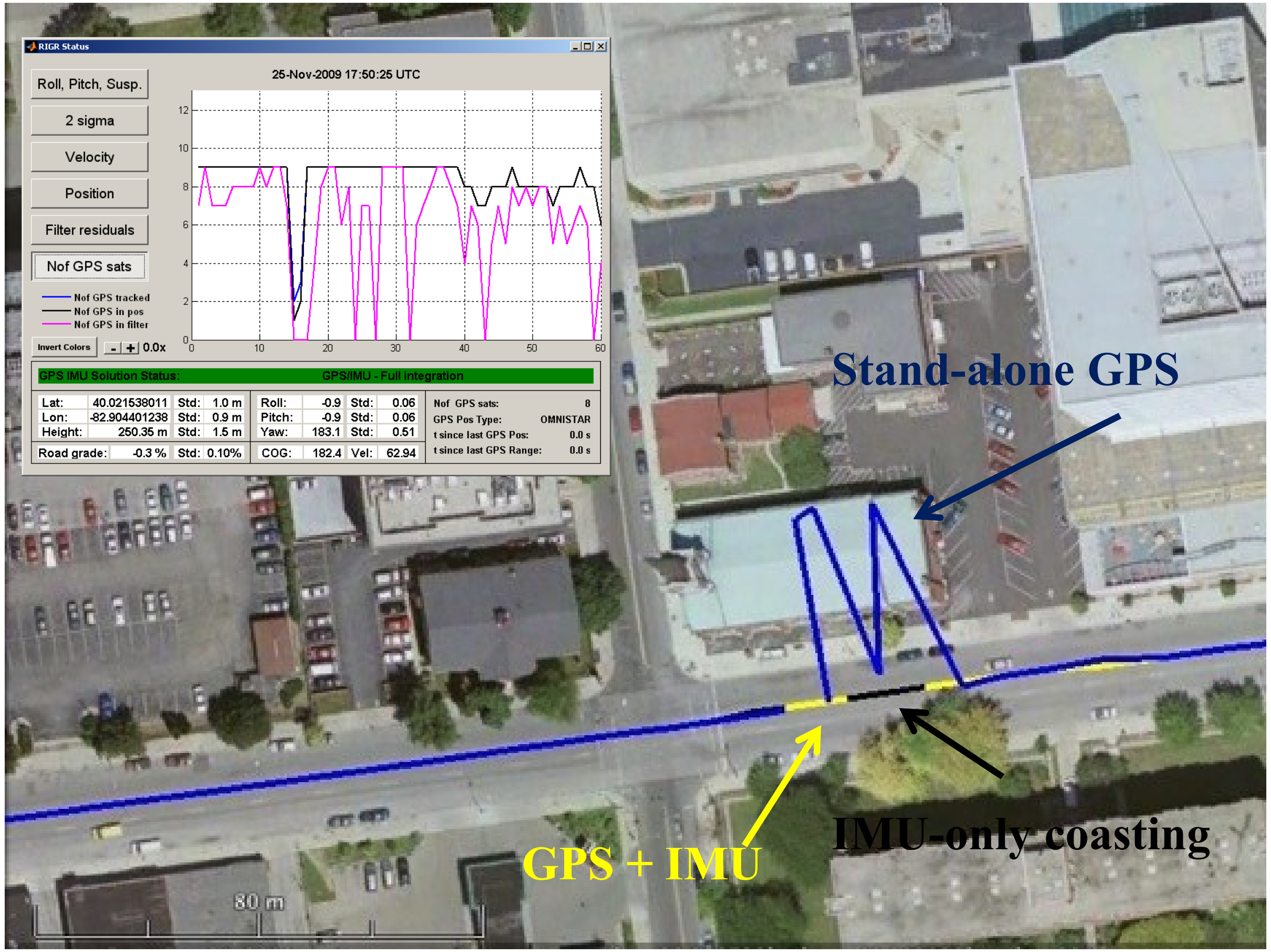
GNSS Limitation

- Most GPS receivers report fragmented satellite availability in urban areas
- Satellite availability with NovAtel OEM-4 GPS receiver in a small USA town



SV = space vehicle = satellite

Use of GNSS in Urban Environments



GPS:
- **Limited satellite visibility;**
- **Multipath;**
- **Fading;**
- **Indirect-only Signal Tracking**
- **Intermittent Tracking**

Current Augmentations:
- **Integration with maps;**
- **Integration with odometers;**
- **Integration with INS**

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Examples






GNSS Vulnerabilities

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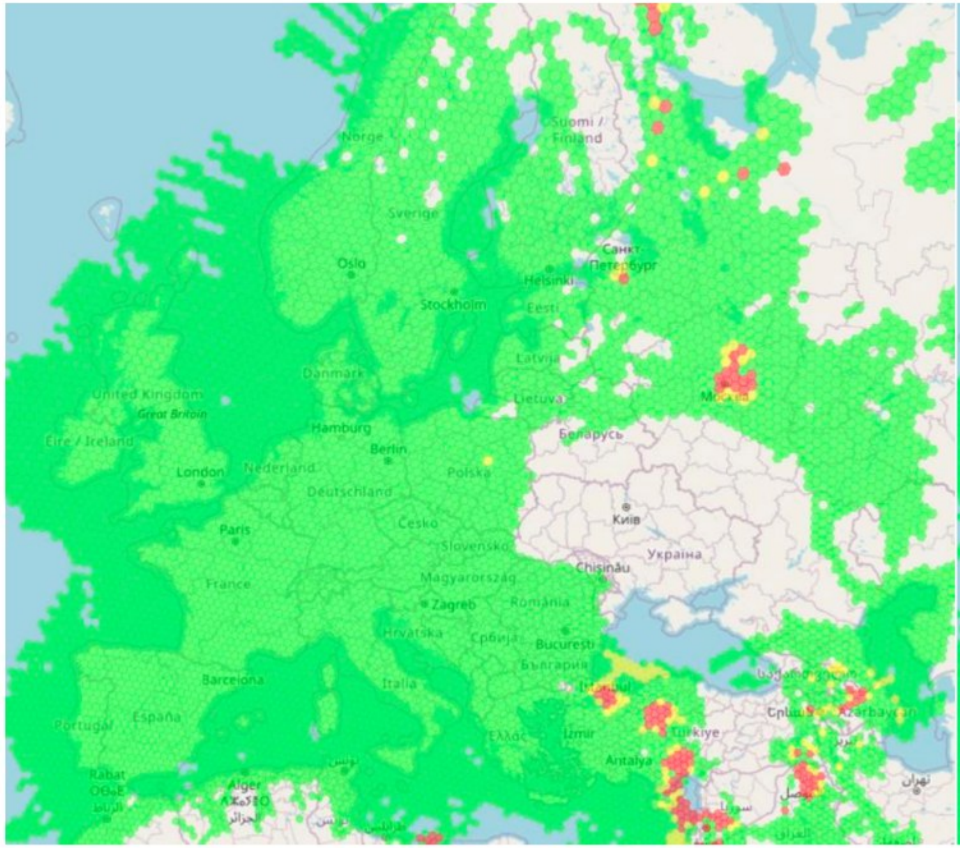
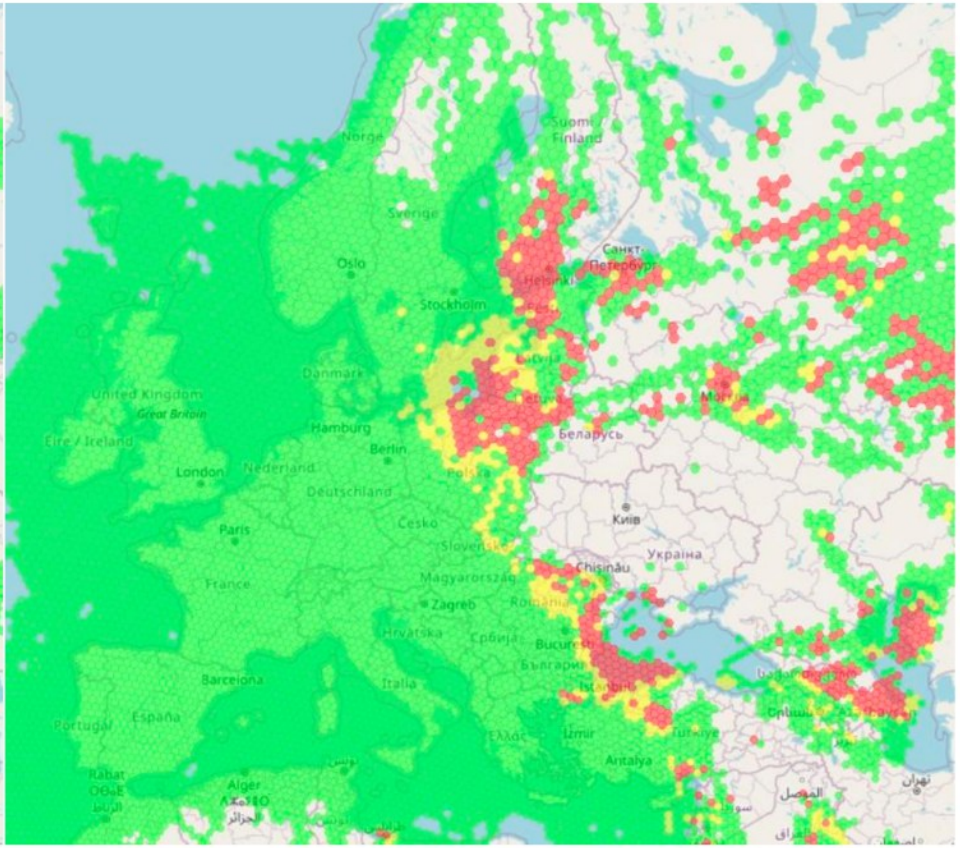
3 years ago (9/07/2022)



TODAY (9/07/2025)



Source: Gpsjam.org

GNSS Vulnerabilities

"Russia blamed for GPS interference affecting flights in Europe"

"Planes Hit by Mystery GPS Jamming Across Europe"

"GNSS outage leading to navigation surveillance degradation"

- There has been an increase in jamming and spoofing of GNSS
- Can result in:
  - inconsistent flight guidance,
  - loss/misleading ADS-B, TAWS, clock info
  - Inconsistent information on navigation display
  - Inability to use GNSS for navigation, including waypoint navigation;
  - Inability to conduct or maintain GNSS based RNAV / RNP operations.

See EASA SIB

Courtesy of O. Osechas

Sensor Fusion for Assured Navigation

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Goal:

- Combine (integrate or **fuse**) data from multiple sensors (or navigation aids) in such a way that the Required Navigation Performance of the intended operation can be met (**assured navigation**).

Objective of a navigation system is to provide an accurate Position, Velocity, Attitude, and Time (PVAT) expressed in the coordinates of some geometric reference: Position, Velocity, Attitude, Time

↓

Required Navigation Performance for Intended Operation

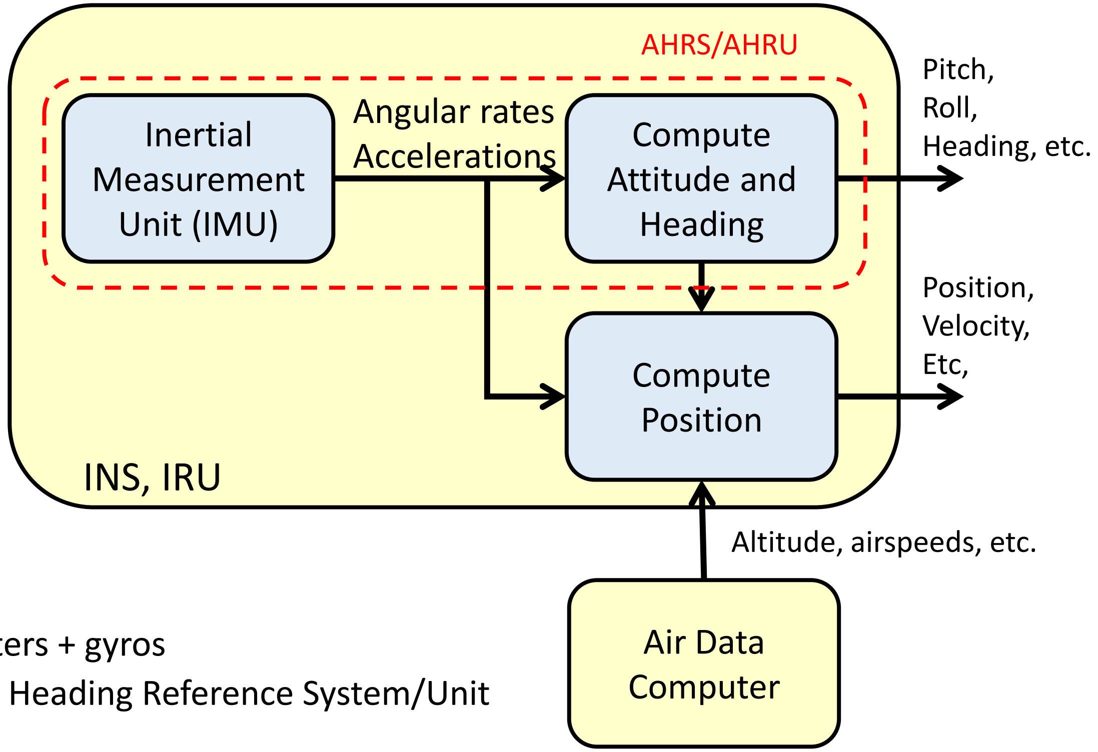
Accuracy, Integrity, Availability, Continuity, etc.

Potential Solutions to Achieve Performance

- **Robust GPS Precision Navigation**:
  - Integration (tight, ultra-tight) of INS with GPS
  - Integration with other dead-reckoning sensors
- **Alternative Navigation Methods**:
  - Signals-of-Opportunity - (LEOs like Starlink, radio, TV, WiFi, cell-phones)
  - Image/Laser-aided Inertial Navigation
  - Beacon-based Navigation
  - Precision INS and MEMS
- **Multiple-sensor/vehicle integration**:
  - Multi-aperture (N-view geometry) using multiple vehicles
  - Cooperative navigation of multiple vehicles

Inertial Navigation Systems (INS)

- Inertial Navigation System (INS) consists of a:

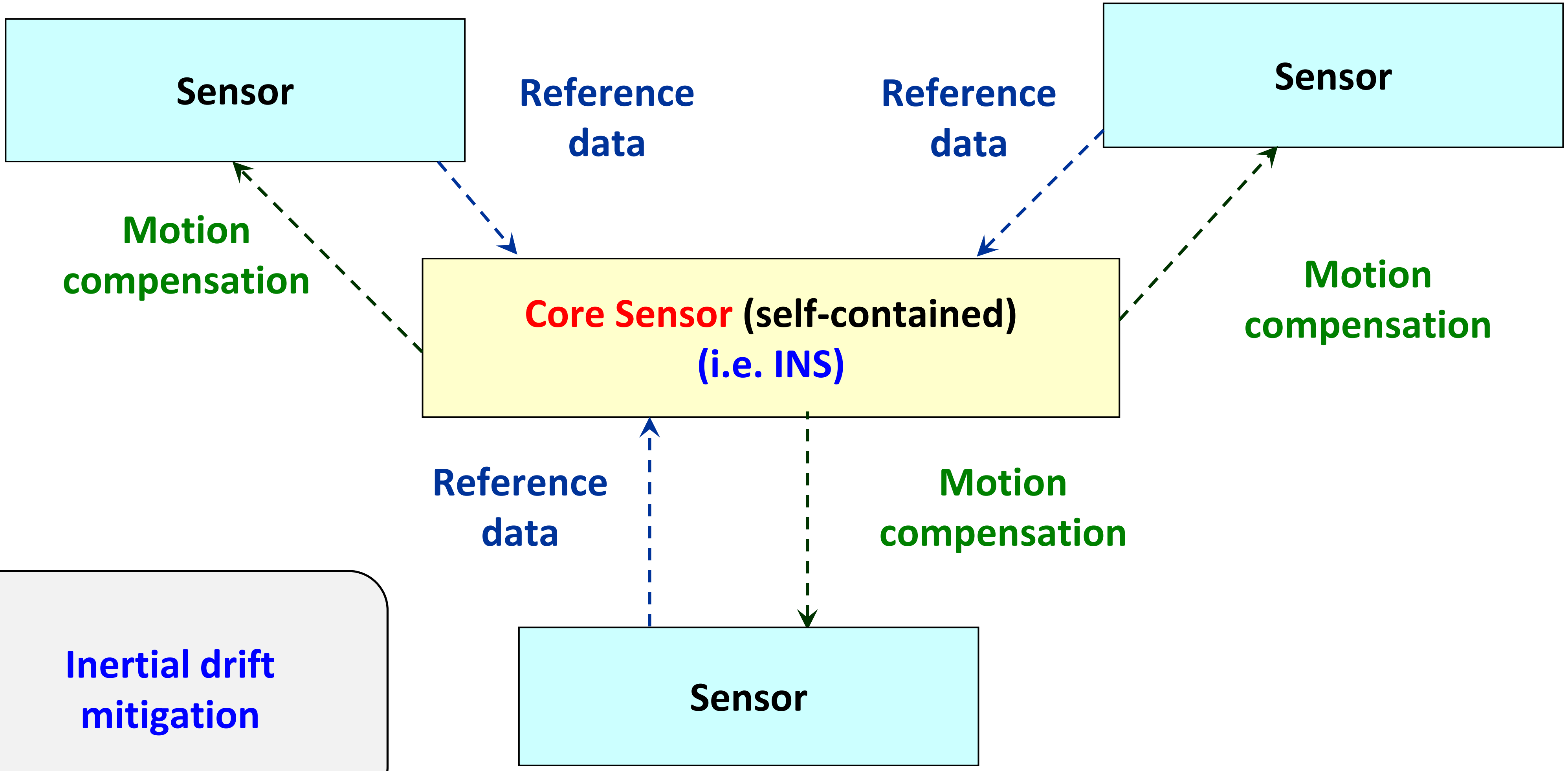


IMU = triad of accelerometers + gyros

AHRS/AHRU = Attitude and Heading Reference System/Unit

- A typical INS position error grows as a function of time – a.k.a. inertial drift!!!

Sensor Integration with Inertial



Inertial drift mitigation

Example: GNSS/INS

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GNSS	INS
Excellent long-term accuracy (stable)	Relatively poor long-term accuracy; errors increase over time
Somewhat noisy in the short term	Relatively noise-less in the short term
Relatively low data rates (~1-20Hz)	Relatively high data rates (~50-3600Hz)
Relatively long data latencies (0.1-1 second)	Relatively short data latencies
Requires a special setup to determine attitude	High quality attitude

Complementary in Nature

- **Issues**:
  - Asynchronous data streams
  - Ability to detect and isolate satellite (SV) failures,
  - Robust operation with fewer then 4 satellites available
  - Lever arm between GPS and INS

Novatel SPAN

VectorNAV

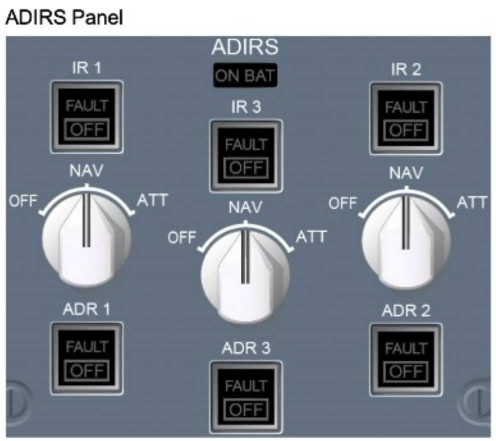
XSense

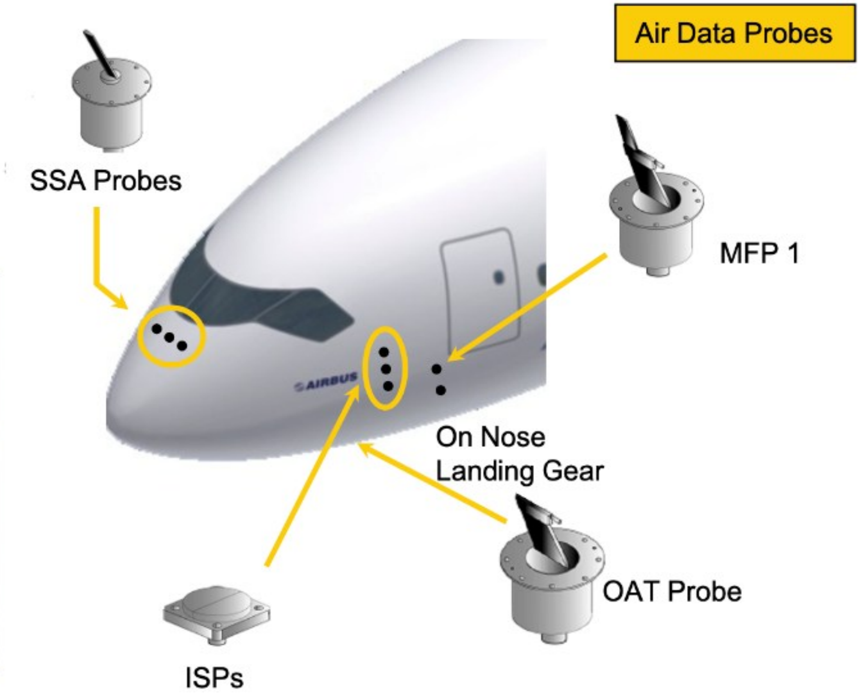
Airbus (based on A350) – ADIRS

Air Data and Inertial Reference System (ADIRS)					
ADIRU		ADIRU		ADIRU	
ADR	IR	ADR	IR	ADR	IR
1 MFP, 1 SSA, 2 ISPs	*Attitude, position (lat/lon), velocity, heading etc.*	*1 MFP, 1 SSA, 2 ISPs*	*Attitude, position (lat/lon), velocity, heading etc.*	*1 MFP, 1 SSA, 2 ISPs*	*Attitude, position (lat/lon), velocity, heading etc.*
In addition: fourth AOA probe provides additional angle of attack measurements, and two Outside Air Temperature (OAT) probes provide Static Air Temperature (SAT) on ground only					

- **Multi-Function Probe (MFP)**: provides total pressure, Total Air Temperature and Angle of Attack measurements
- **Side Slip Angle (SSA)**: provides the sideslip angle
- **Integrated Static Probe (ISP)**: provides the static pressure

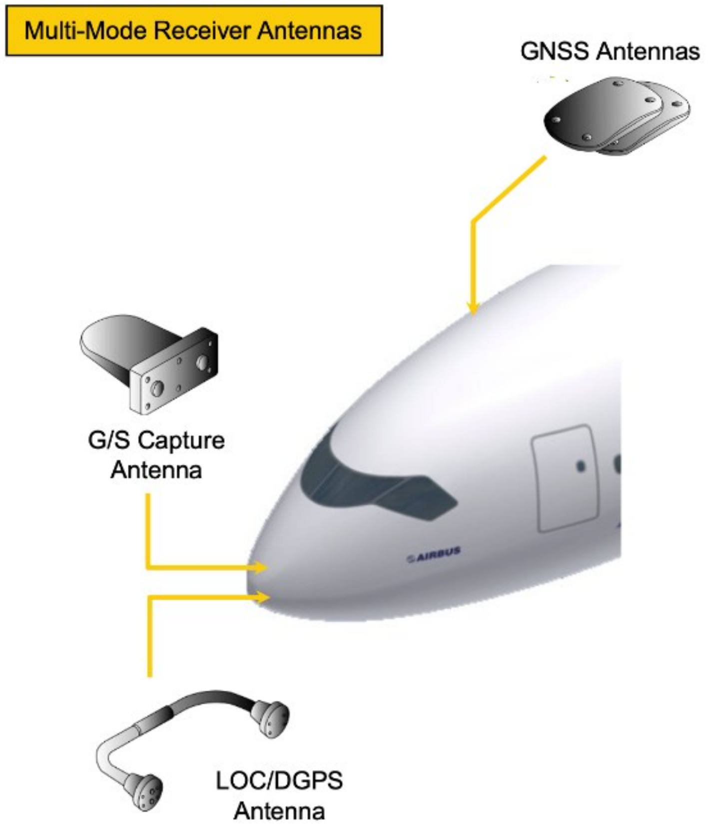
ADIRU = Air Data and Inertial Reference Units
ADR = Air Data Reference
IR = Inertial Reference





Airbus (based on A350) – MMR – GPS

- MMR (Multi-Mode Receiver) System
  - 2 MMRs
  - 2 Global Navigation Satellite System (GNSS) antennas
  - One Localizer / Differential Global Positioning System (LOC/DGPS) antenna
  - One glide slope capture antenna.
- MMR Functions:
  - Landing systems:
    - Instrument Landing System (ILS)
    - FMS Landing System (FLS)
    - **Ground Based Augmentation System (GBAS) Landing System (GLS)**
    - Satellite Landing System (SLS)
  - Navigation Systems:
    - Global Navigation Satellite System (GNSS) - GPS



Airbus (based on A350) – Aircraft Position Calculation

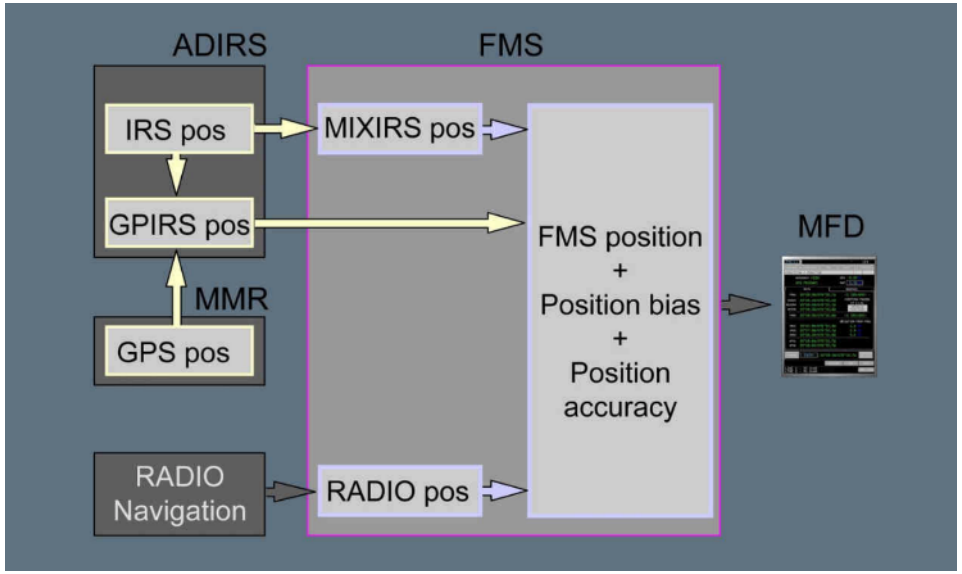
- **Hybrid Position**
  - based on a combination of inertial and GPS data (1 IRS/GPS source).
- **Consolidated Hybrid Position**
  - based on a combination of all hybrid positions (3 IRS/GPS sources).

To improve Accuracy, Integrity, Availability, and Continuity

- **FMS Position**
  - each FMS computes its aircraft position and the position accuracy, using inertial via ADIRS, GPS via MMR, and radio navigation via NAVAIDS* receivers
  - Inertial GPS (IRS/GPS)
  - Inertial – DME/DME (IRS/DME/DME)
  - Inertial – VOR/DME (IRS/VOR/DME)
  - Inertial only (IRS)

Decreasing order of priority

- **Pure INS Position**
  - when only the IRS sources are available

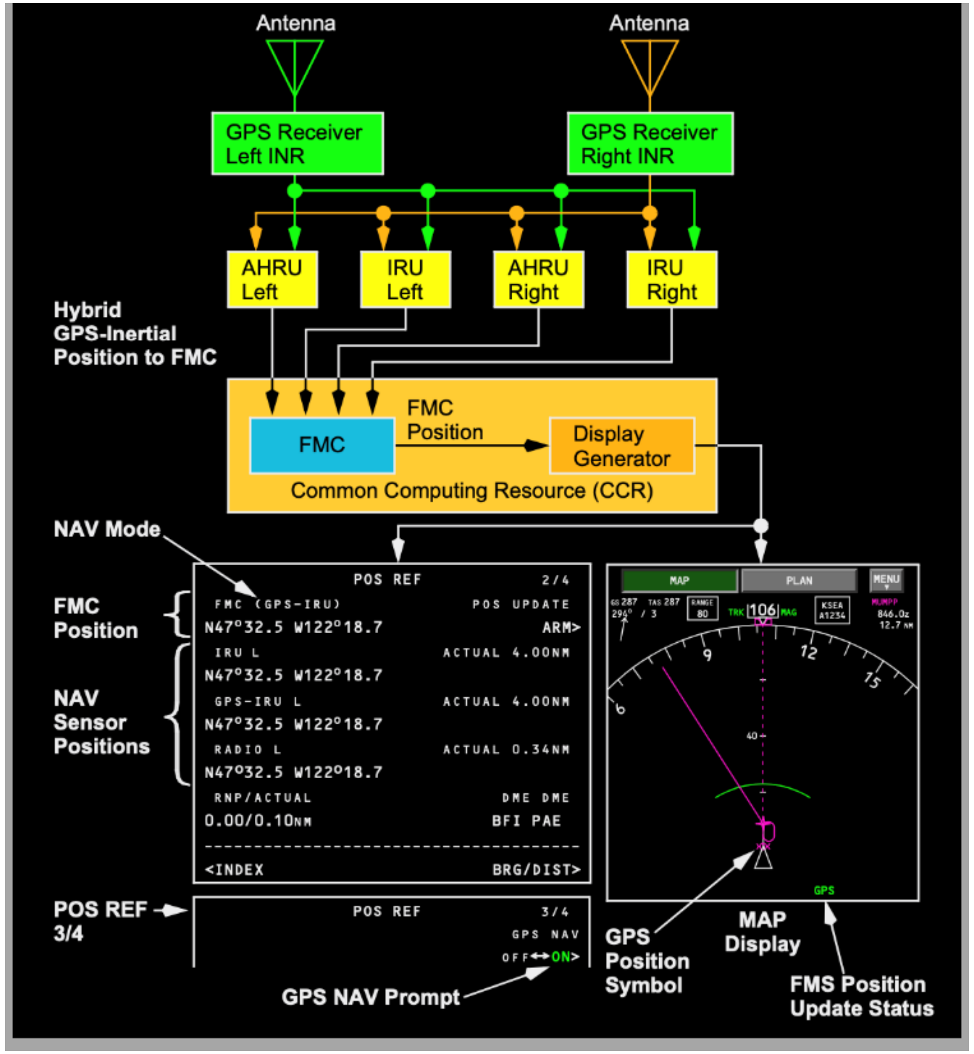


From A380 FCOM

*VOR, DME, etc.

Boeing (based on B787)

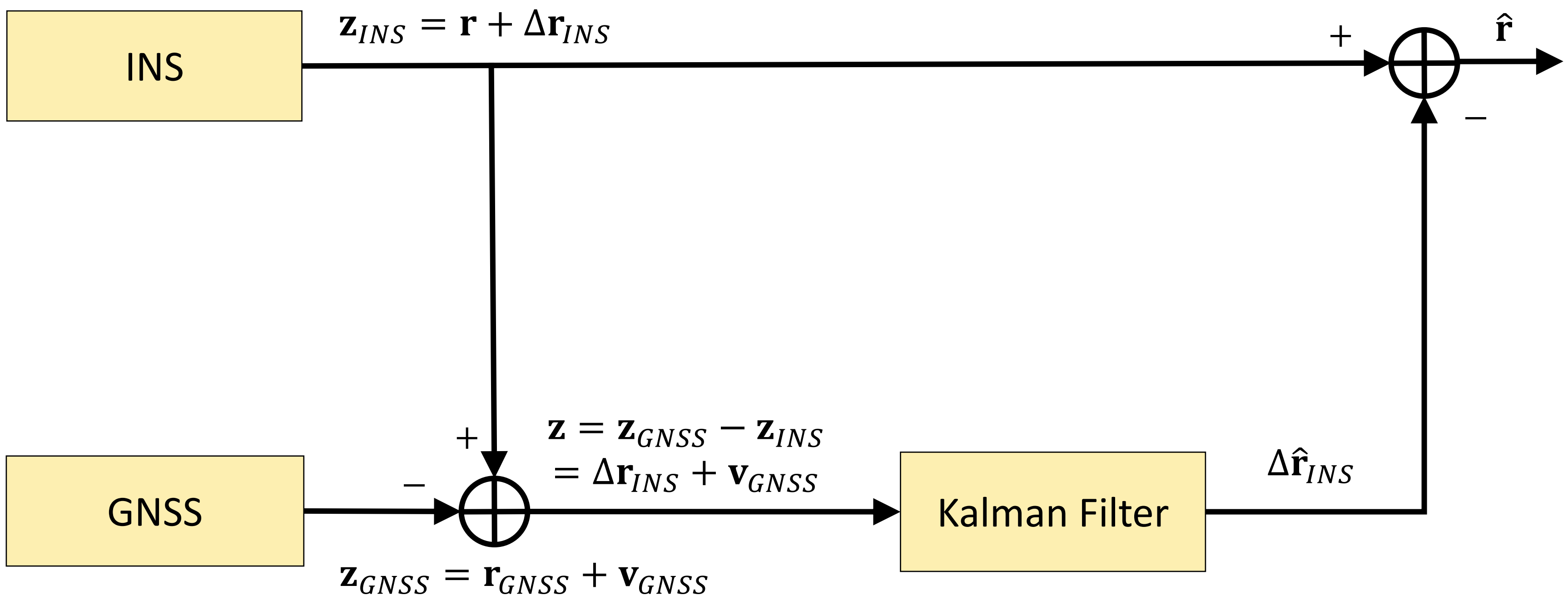
- Integrated navigation radios (INRs)
  - Two pieces
  - GPS/GBAS etc.
- Inertial Reference System (IRS)
  - 2 inertial reference units (IRU)
    - Position, velocity, attitude, heading
  - 2 attitude heading reference units (AHRU)
    - Attitude, heading
- Air Data Reference System (ADRS)
  - 6 air data modules
  - 6 static ports (3 on the left, 3 on the right)
  - 3 pitot probes (left, right, center)
  - 2 angle of attack vanes (left, right)
  - 1 total air temperature (TAT) probe
    - also receives TAT measurements from each of the EEC TAT sensors for a total of six inputs



Hybrid GPS/IRS

✈ Loose integration

- Commonly performed in the position domain: take position output from inertial and combine with position output from GPS (from a least squares solution for example)



Position estimates from GNSS receiver (calculated using OLS or WLS)

Pick a coordinate frame to implement in, for example ECEF or LLH

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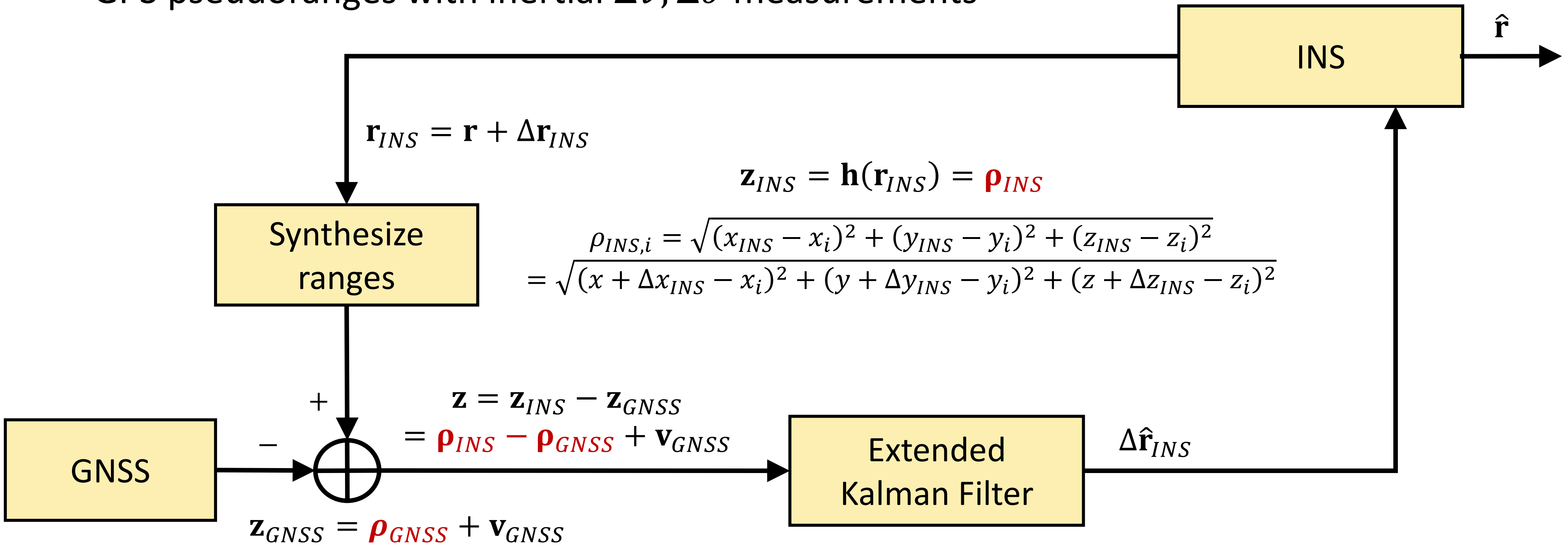
Tight integration

- Commonly performed in the measurement domain:
  - GPS pseudoranges with inertial position
  - GPS pseudoranges with inertial $\Delta v, \Delta\theta$ measurements



Pseudorange measurements from GNSS

Pick a coordinate frame to implement in, for example ECEF

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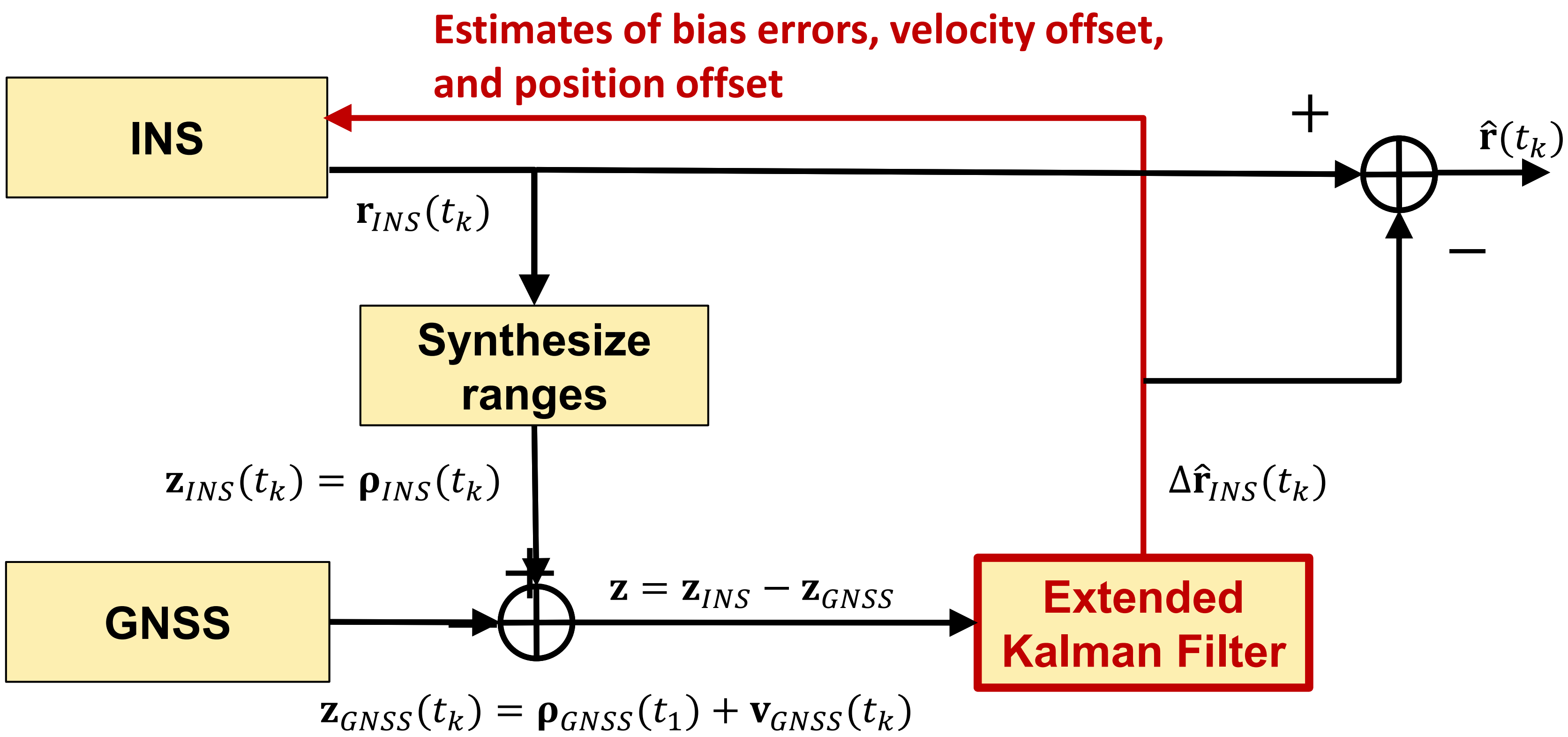
Tight coupling with feedback using a complementary Kalman Filter



Various Hybridization Methods

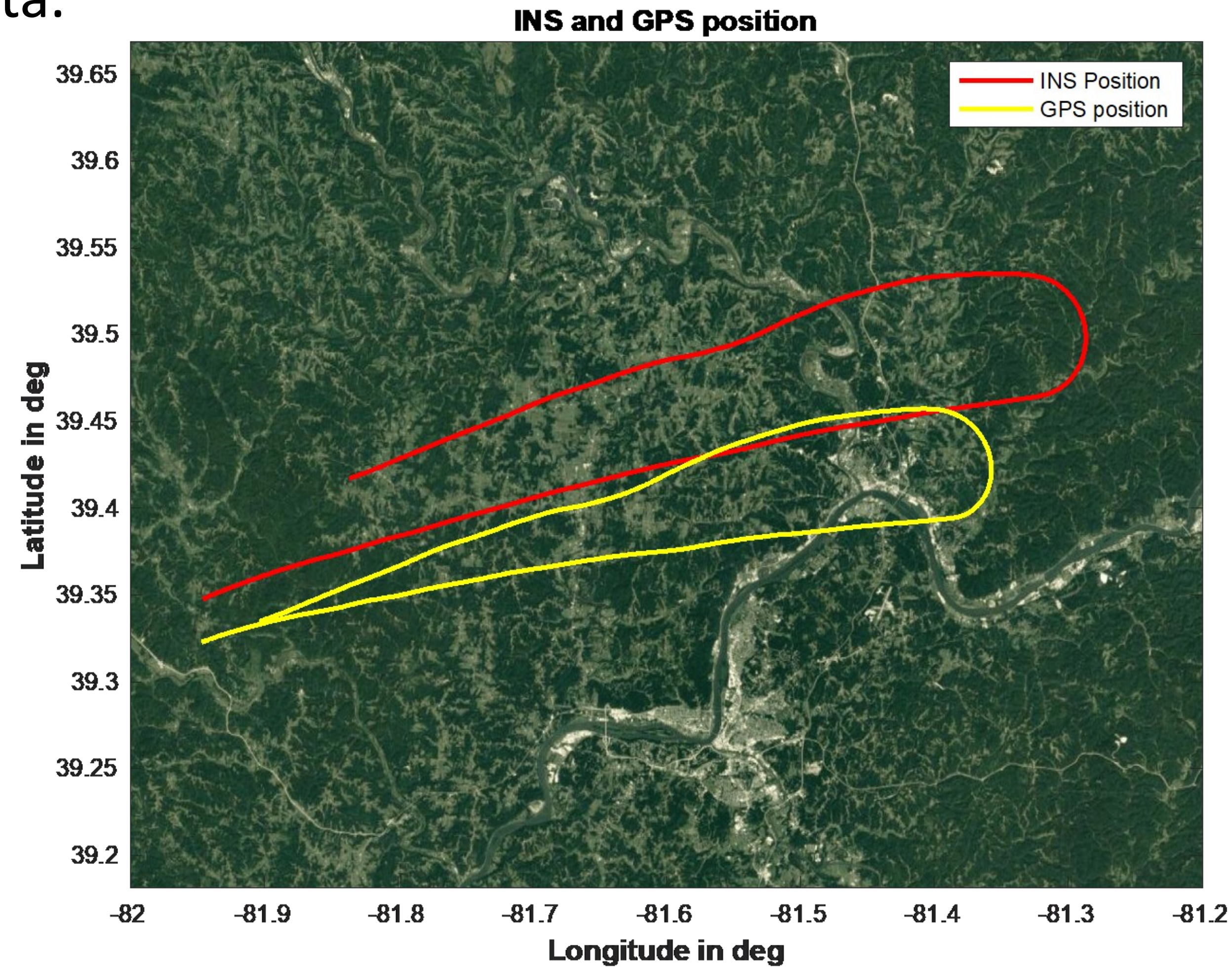
GNSS	INS	Integration Level	Example state vectors
$\mathbf{r}_{GNSS}$	$\mathbf{r}_{INS}$	Loose coupling	$[\delta\mathbf{r}_k \quad \delta\dot{\mathbf{r}}_k]^T$
$\dot{\mathbf{r}}_{GNSS}$	$\dot{\mathbf{r}}_{INS}$	Loose coupling*	$[\delta\mathbf{r}_k \quad \delta\dot{\mathbf{r}}_k]^T$
$\mathbf{r}_{GNSS}, \dot{\mathbf{r}}_{GNSS}$	$\mathbf{r}_{INS}, \dot{\mathbf{r}}_{INS}$	Loose coupling	$[\delta\mathbf{r}_k \quad \delta\dot{\mathbf{r}}_k]^T$
$PR_{GNSS,j}$	$\mathbf{r}_{INS}$	Tight coupling	$[\delta\mathbf{r}_k \quad \delta\dot{\mathbf{r}}_k \quad \delta b_k \quad \delta\dot{b}_k]^T$
$PR_{smooth,j}$	$\mathbf{r}_{INS}$	Tight coupling	$[\delta\mathbf{r}_k \quad \delta\dot{\mathbf{r}}_k \quad \delta b_k \quad \delta\dot{b}_k]^T$
$\Delta\phi_j^{comp+adj}$	$\Delta\mathbf{r}_{INS}$	Tight coupling	$[\delta\mathbf{r}_k \quad \delta\dot{\mathbf{r}}_k \quad \boldsymbol{\psi} \quad \delta\boldsymbol{\omega}^b \quad \delta\mathbf{f}^b \quad \delta b_k \quad \delta\dot{b}_k]^T$
$PR_{GNSS,j}, \Delta\phi_j$	$\Delta\mathbf{r}_{INS}$	Tight coupling (combined)	$[\delta\mathbf{r}_k \quad \delta\dot{\mathbf{r}}_k \quad \boldsymbol{\psi} \quad \delta\boldsymbol{\omega}^b \quad \delta\mathbf{f}^b \quad \delta b_k \quad \delta\dot{b}_k]^T$
Dynamics: $\Delta\phi_j$ Position: $PR_{GNSS,j}$,	$\Delta\mathbf{r}_{INS}$	Tight coupling (separate)	Dynamics: $[\delta\dot{\mathbf{r}}_k \quad \boldsymbol{\psi} \quad \delta\boldsymbol{\omega}^b \quad \delta\mathbf{f}^b \quad \delta b_k \quad \delta\dot{b}_k]^T$ Position: $[\delta\mathbf{r}_k \quad \delta\dot{\mathbf{r}}_k]^T$

*Requires feedback

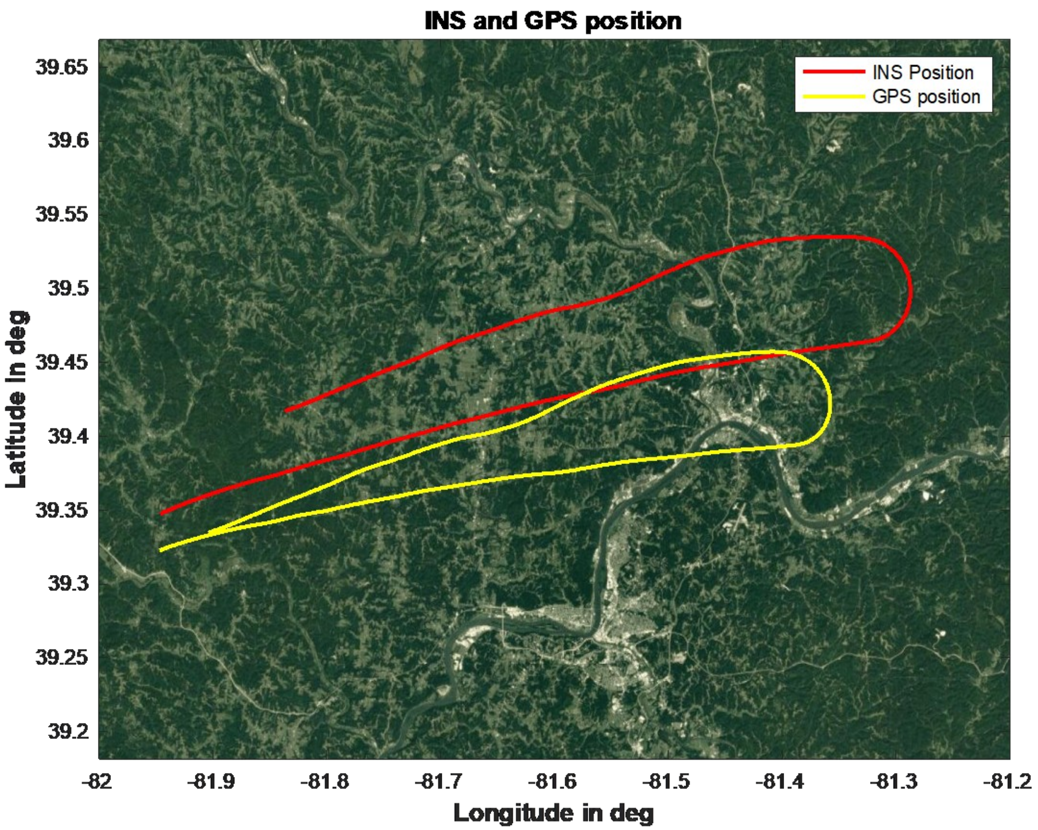
$\dot{\mathbf{r}}_{GNSS}$ from precise velocity

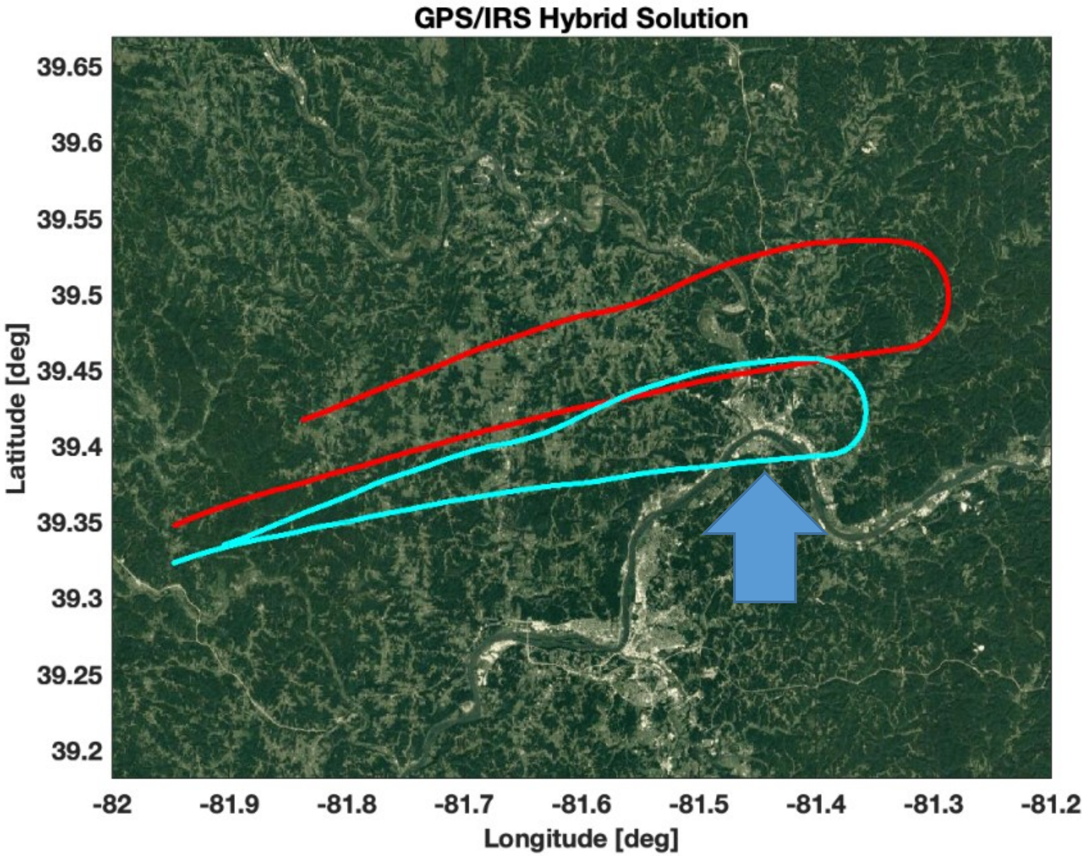
Example – GPS and INS standalone

- Integration of GNSS (GPS) and data from a Navigation-grade certified Inertial Navigation System (INS) using some real flight test data.



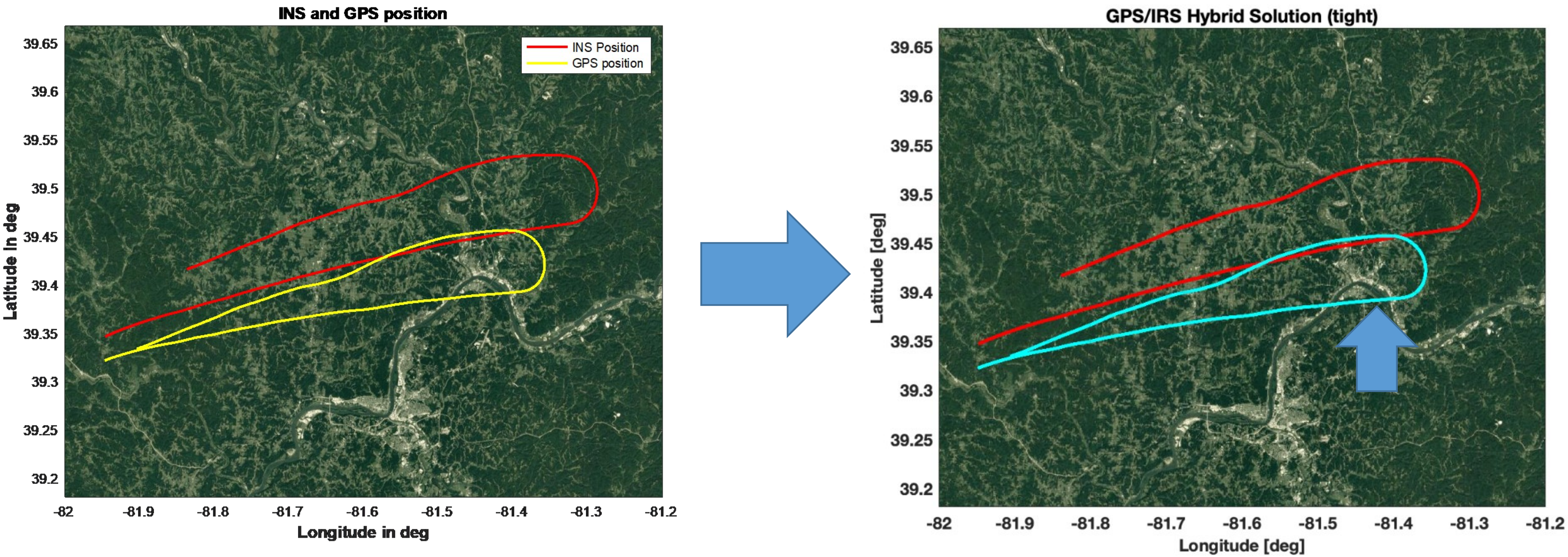
Example – GPS/INS Hybrid Solution (Loose Integration)





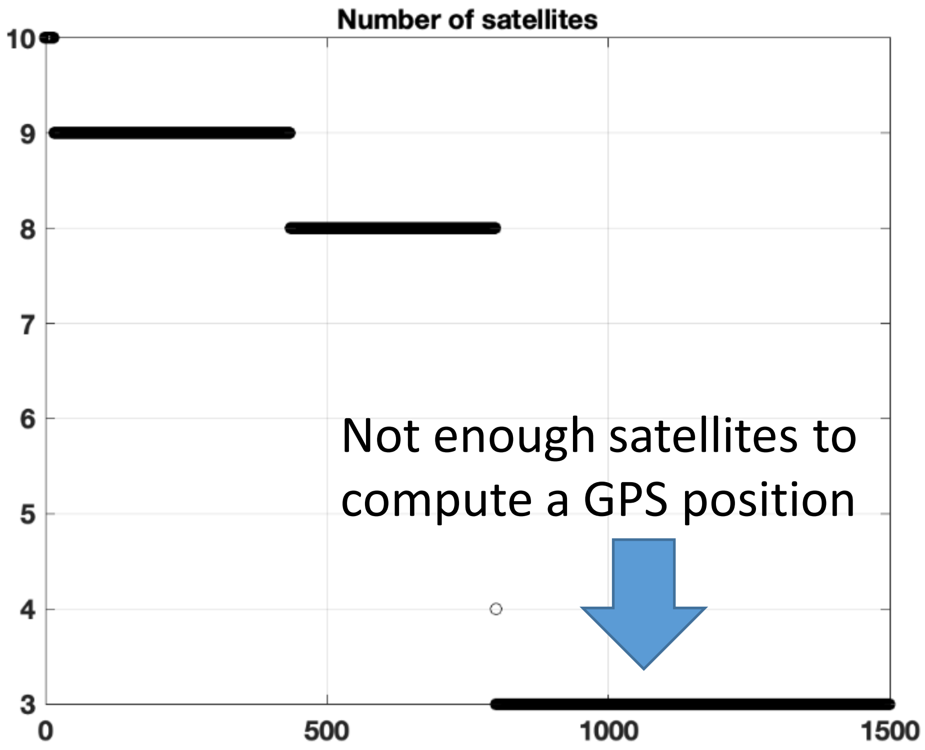
- The hybrid solution (loose) overlays the GPS solution , but is less noisy than the standalone GPS solution; it combines the smoothness of the INS trajectory with the non-drifting behavior of GPS

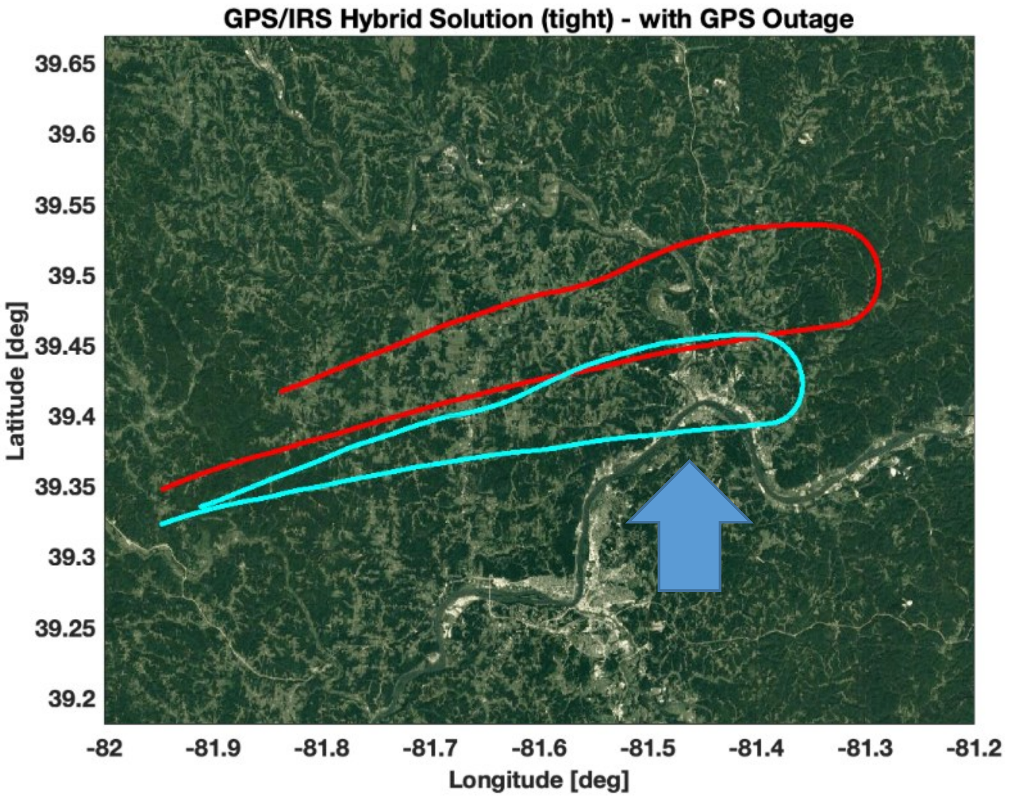
Example – GPS/INS Hybrid Solution (Tight Integration)



- ✈ The hybrid solution (tight) overlays the GPS solution , but is less noisy than the standalone GPS solution; it combines the smoothness of the INS trajectory with the non-drifting behavior of GPS

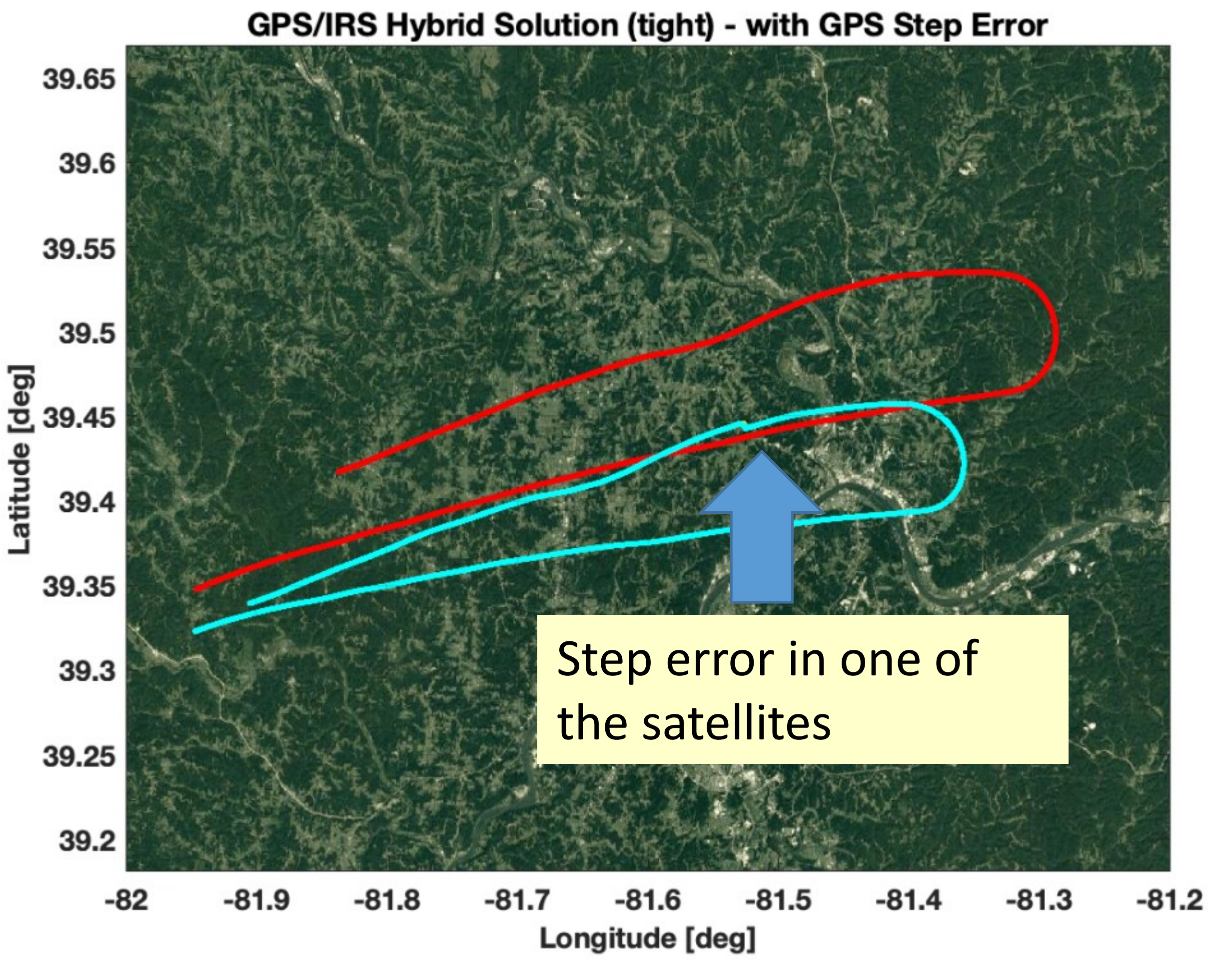
Example – GPS/INS Hybrid Solution (with Outage)

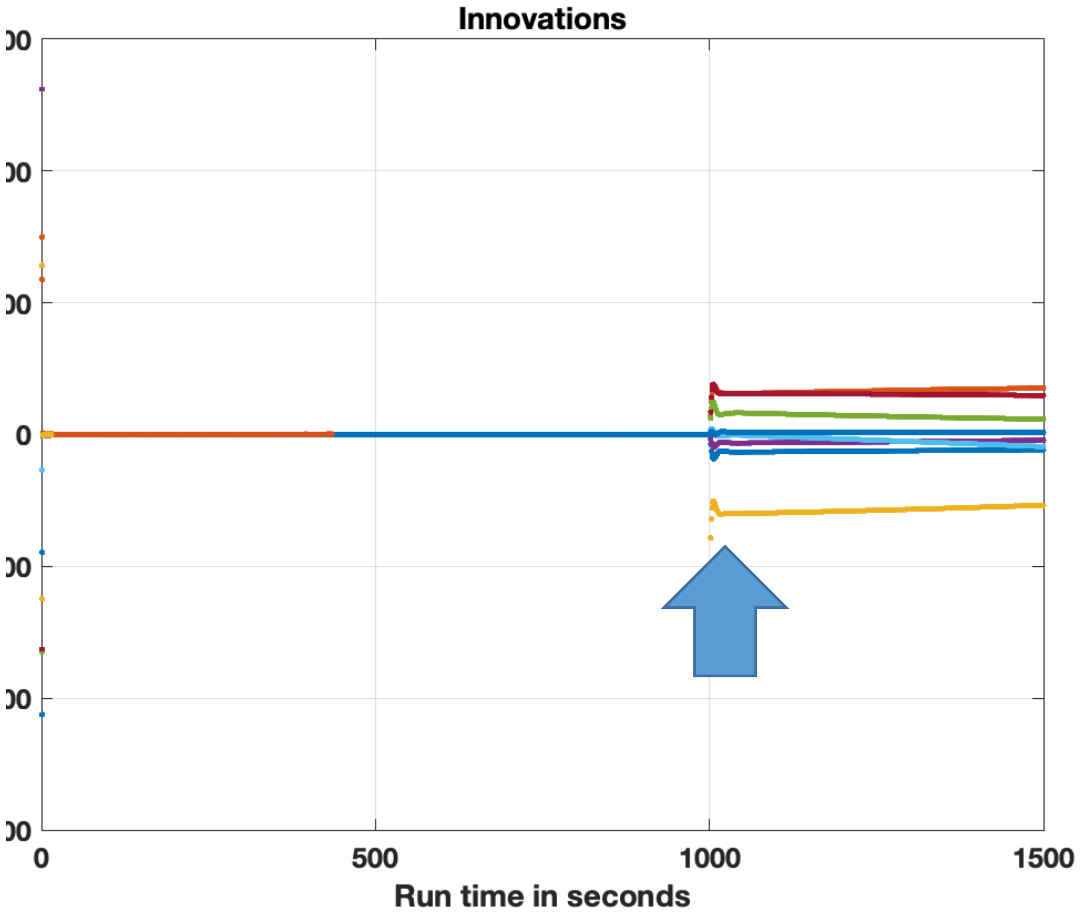




✈ The Hybrid solution (tight) provides continuous position estimates even where the number of satellites falls below 4 and GPS alone would not be available

Example – GPS/INS Hybrid Solution (with Step Error)

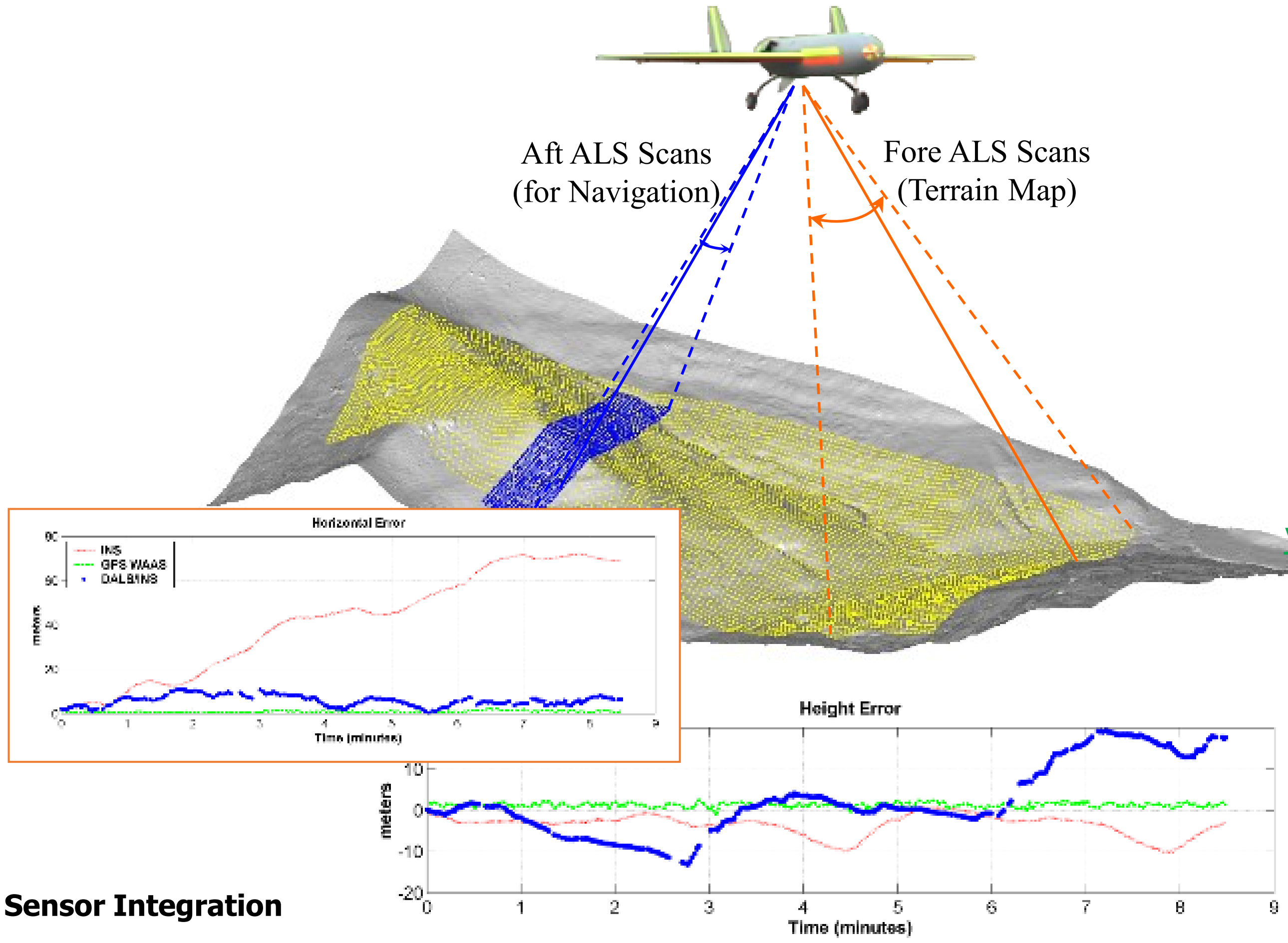


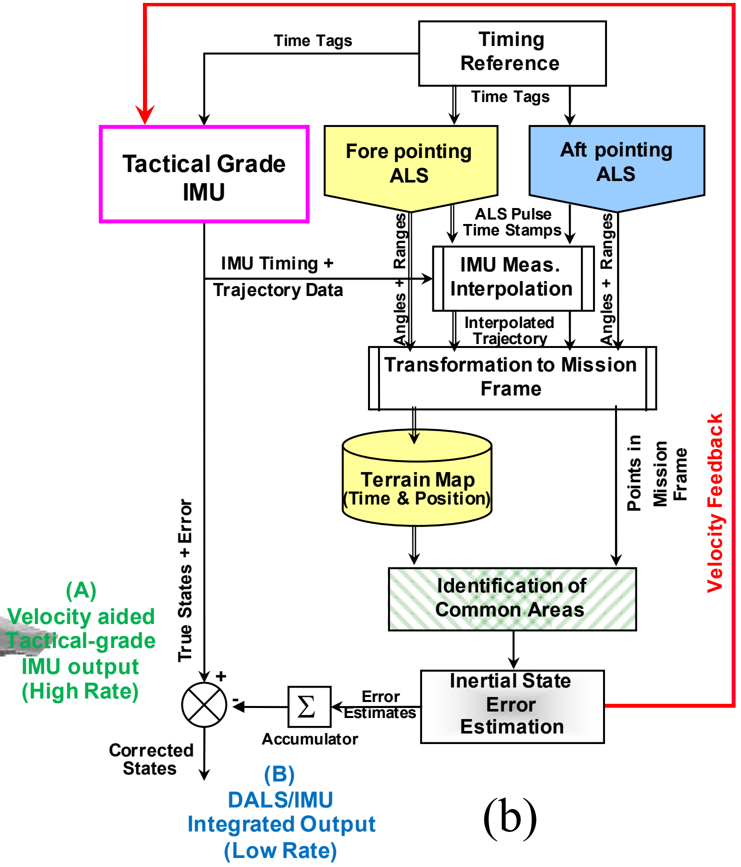


- ✈ Step errors can be observed in the innovations

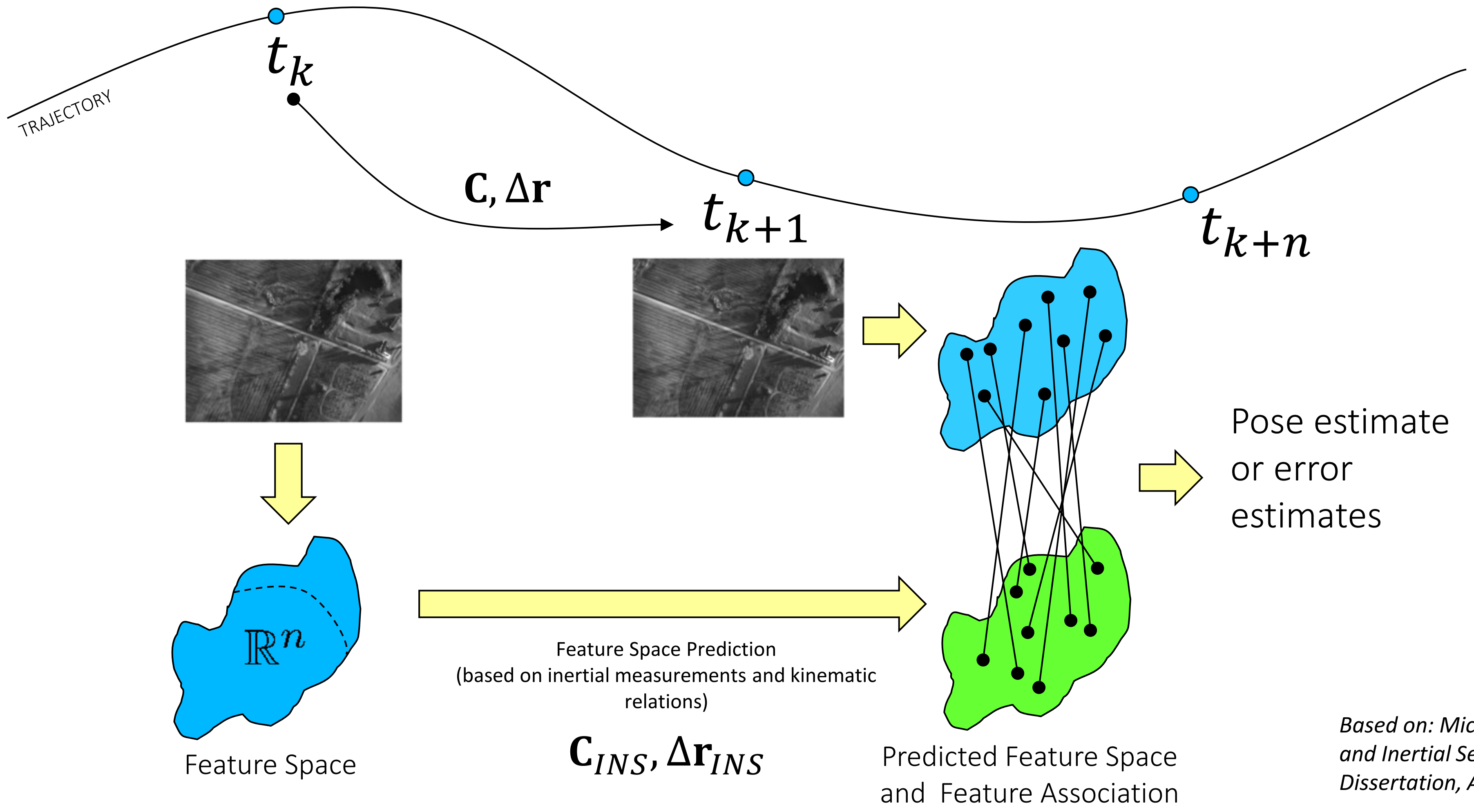
Laser-based Navigation

- Using (i) features, (ii) maps or occupancy grids, maps, (iii) scan matching, etc.
- Integration approach: loose, tight with IMU or SLAM



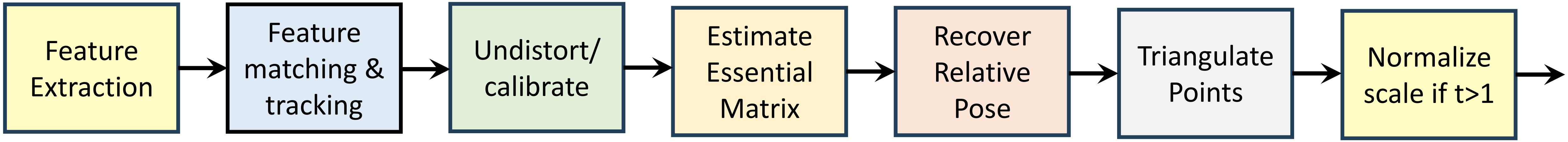


Vision-based Navigation



Based on: Michael J. Veth, "Fusion of Imaging and Inertial Sensors for Navigation"," Ph.D. Dissertation, AFIT, 2006.

Vision-based Navigation – VO, VIO, V-SLAM



- Direct versus indirect methods to estimate pose and maps
  - **Direct**: photometric information from the image pixels
  - **Indirect**: pre-process the image and use extracted features or optical flow vectors
- Dense, sparse and semi-dense methods:
  - **Dense**: use all image points
  - **Sparse**: use only a selected subset of point
  - **Semi-dense**: use a large subset of points

VO: Visual Odometry
VIO: Visual Inertial Odometry
V-SLAM: Visual SLAM